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In earlier classes, we have study the statistical and classical approach of probability. But both the theories have some serious difficulties and limitations. To overcome these difficulties, modern probability theory has been developed axiomatically. This theory of probability was developed by Russian Mathematician A.N. Kolmogorov in 1933. In his book "Foundation of probability", he laid down some axioms to interpret probability. To understand this approach, we must know about some basic terms, viz. experiment, random experiment, sample space, events, etc.

PROBABILITY

|TOPIC 1|
Event and Algebra of Events

EXPERIMENT

An operation which can produce some well-defined outcomes, is known as experiment. There are two types of experiments, *viz.*

(i) **Deterministic Experiments** An experiment which gives the same result every time, when it is repeated under identical conditions, is called deterministic experiments.

(ii) **Random Experiments** In our daily life, we perform many experimental activities, where the result may not be same, when they are repeated under identical conditions. e.g. when a fair coin is tossed it may turn up a head or a tail, but we are not sure which one of these results will be actually obtained. Such experiments are called random experiments. Thus, rolling an unbiased die, drawing a card from a well-shuffled pack of cards are all examples of random experiments.

An experiment is called random experiment, if it satisfies the following two conditions.

(i) It has more than one possible outcomes.

(ii) It is not possible to predict the outcomes in advance.

Outcomes and Sample Space

A possible result of a random experiment is called its outcome. The set of all possible outcomes in a random experiment is called a sample space and it is generally denoted by S, i.e.

Sample space (S) = {All possible outcomes}

CHAPTER CHECKLIST

- Event and Algebra of events
- Axiomatic Approach to Probability
- Addition theorems on Probability

e.g. Suppose a die is thrown and let A be an event of getting an odd number.

Then, $$A = \{1, 3, 5\}$$

Now, if the outcome of experiment is 3. Then, we can say that event A has occurred. Suppose, in another trial, outcome is 4, then we can say that event A has not occurred.

Types of Events

Events can be classified into various types on the basis of their elements.

Sure Event The whole sample space S is also a subset of S, so it represents an event. Since, every outcome of an experiment carried out a member of S, therefore the event represented by S is called a sure or a certain event.

e.g. (i) Sun rises in the East, is a sure event.

(ii) On tossing a coin, either head or tail will occur, is a sure event.

(iii) On throwing a die, we have sample space

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Let $E =$ Event of getting a number less than 7.

Then, $E = \{1, 2, 3, 4, 5, 6\}$.

So, E is a sure event.

Impossible Event The empty set ϕ is also a subset of the sample space (S) so it represents an event. Since there is no element in the empty set, therefore it can not occurs.

That's why event ϕ is called an impossible event.

e.g. On throwing a die, we have the sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let $E =$ Event of getting a number less than 1.

Then, $E = \phi$.

So, E is called an impossible event.

EXAMPLE |3| Identify that following events as sure/impossible events.

(i) Event of getting a number multiple of 7 in a single toss of a die.

(ii) Event of getting five kings in a pack of cards.

(iii) Event of drawing a club queen in a pack of cards.

Sol. (i) We know that in a single toss of a die only 1 to 6 number exist. So, multiple of 7 does not exist.

(ii) We know that in a pack of cards only four kings exist. So five kings does not exist. Hence, it is an impossible event.

(iii) We know that in a pack of cards club queen exist. Hence, it is a sure events.

Simple Event If an event has only one sample point then it is called a simple or elementary event.

e.g. Let a die is thrown, then sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

Again, let $A =$ Event of getting $4 = \{4\}$

$B =$ Event of getting $1 = \{1\}$

Here, A and B are simple events.

Note

Elementary events associated with a random experiment are also known as indecomposable events.

Compound Event If an event has more than one sample point, then it is called a compound event.

e.g. On rolling a die, we have sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

Then, the events

E : Getting an even number

F : Getting an odd number

G : Getting a multiple of 3

are all compound events. The subsets of S associated with these events are

$$E = \{2, 4, 6\},$$
$$F = \{1, 3, 5\}, G = \{3, 6\}$$

Each of the above subsets contain more than one sample point, therefore they are all compound events.

Note

All events other than elementary events and impossible events associated with a random experiment are compound events.

EXAMPLE |4| Two dice are rolled. Let A, B, C be the events of getting a sum of 2, a sum of 3 and a sum of 4 respectively.

(i) Which events are elementary events?

(ii) Which events are compound events?

Sol. On throwing of two dice, we get the sample space

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

Now, as A is the event of getting a sum of 2.

$\therefore$ $A = \{(1,1)\}$

Similarly, $B = \{(1,2),(2,1)\}$

and $C = \{(1,3),(3,1),(2,2)\}$

(i) Since, A consists a single sample point, therefore it is an elementary or simple event.

(ii) Since, both B and C contain more than one sample points, therefore each of them is compound event.

ALGEBRA OF EVENTS

We know that how to combine two or more sets by using the operations on sets union ($\cup$), intersection ($\cap$)and difference ($-$). Like-wise we, can combine two or more events by using the analogous set notations. Let A and B are two events associated with a sample space S, then

1. COMPLEMENTARY EVENT

For every event E, there corresponds another event E' called the complementary event of E, which consist of those outcomes that do not correspond to the occurrence of E. E' is also called the event 'not E'.

e.g. In tossing three coins, the sample space is

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Let $E = \{THT, TTH, HTT\}$

= the event of getting only one head.

Clearly, for outcome HHT, event E has not occurred

$\therefore \quad HHT \in E'$

Similarly, for the outcomes HTH, THH, TTT and HHH, event E has not occurred.

$\therefore \quad E' = \{HHT, HTH, THH, TTT, HHH\}$

It can be seen that E' contains only those outcomes which are not in E.

EXAMPLE |5| Two dice are thrown once. The events A, B, E are as follows

A : Getting an even number on first die.

B : Getting an odd number on first die.

E : Getting the sum of numbers on the dice ≥ 10.

Describe the events

(i) A' (ii) B' (iii) E' **[NCERT]**

Sol. On throwing of two dice, we have sample space

$$S = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

(i) A : Getting an even number on first die.

$\Rightarrow A'$: Getting an odd number on first die.

$$= \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \end{Bmatrix}$$

(ii) B : Getting an odd number on first die.

$\Rightarrow B'$: Getting an even number on first die.

$$= \begin{Bmatrix} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

(iii) E : Getting the sum of the numbers on the dice ≥ 10.

$\Rightarrow E'$: Getting the sum of the numbers on the dice < 10.

$$= \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), \\ (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), \\ (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), \\ (5,1), (5,2), (5,3), (5,4), (6,1), (6,2), (6,3) \end{Bmatrix}$$

2. THE EVENT '*A* OR *B*'

The event 'A or B' is same as the event $A \cup B$ and it contains all those elements which are either in event A or in events B or in both.

Thus, A or $B = A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$

3. THE EVENT '*A* AND *B*'

The event 'A and B' is same as the event '$A \cap B$' and it contains all those elements which are common to both A and B. Thus, A and $B = A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$

4. THE EVENT '*A* BUT NOT *B*'

The events 'A but not B' is same as the event $A - B = (A \cap B')$ and it contains all those elements which are in A but not in B.

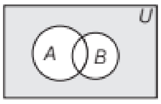
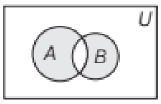
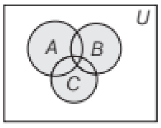
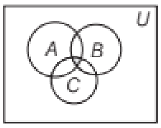
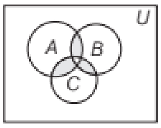
Thus, A but not $B = A - B = \{\omega : \omega \in A \text{ and } \omega \notin B\}$

Note

For any event E, associated with a sample space S,

$$E' = \text{not } E = S - E = \{\omega : \omega \in S \text{ and } \omega \notin E\}.$$

In the following table, we give verbal description of some events and their equivalent set alongwith the venn diagram.

	Events	Verbal Description	Equivalent Set	Venn Diagram
(i)	Neither A nor B	It is the intersection of sets $\overline{A}$ and $\overline{B}$, which contains the common elements of $\overline{A}$ and $\overline{B}$.	$\overline{A} \cap \overline{B}$ or $U - (A \cup B)$	
(ii)	Exactly one of A and B	It is the union of one event and complement of other events. or It is the difference of union and intersection of both events.	$(A \cap \overline{B}) \cup (\overline{A} \cap B)$ or $(A \cup B) - (A \cap B)$	
(iii)	Atleast one of A, B or C	It is a union of all three sets, which contains either A or B or C etc.	$A \cup B \cup C$	
(iv)	All three of A, B and C	It is an intersection of all three sets, which contains common elements of A, B and C.	$A \cap B \cap C$	
(v)	Exactly two of A, B and C	It is the union of three conditions, where in each condition we take intersection of any two events and complement of remaining third event.	$(A \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap C) \cup (\overline{A} \cap B \cap C)$	

EXAMPLE |6| A die is thrown. Describe the following events

(i) A : a number less than 7.
(ii) B : a number greater than 7.
(iii) C : a multiple of 3.
(iv) D : a number less than 4.
(v) E : an even number greater than 4.
(vi) F : a number not less than 3.

Also, find $A \cup B, A \cap B, B \cup C, E \cap F, D \cap E, A - C, D - E, F'$ and $E \cap F'$. **[NCERT]**

Sol. When a die is thrown, then sample space

$S = \{1, 2, 3, 4, 5, 6\}$

(i) A : a number less than 7 = $\{1, 2, 3, 4, 5, 6\}$
(ii) B : a number greater than 7 = $\{\ \} = \phi$
(iii) C : a multiple of 3 = $\{3, 6\}$
(iv) D : a number less than 4 = $\{1, 2, 3\}$
(v) E : an even number greater than 4 = $\{6\}$
(vi) F : a number not less than 3 = $\{3, 4, 5, 6\}$

Now, $A \cup B$ = The elements which are in A or B or both

$= \{1, 2, 3, 4, 5, 6\} \cup \phi$
$= \{1, 2, 3, 4, 5, 6\}$

$A \cap B$ = The elements which are common in both A and B

$= \{1, 2, 3, 4, 5, 6\} \cap \phi = \phi$

$B \cup C$ = The elements which are in B or C or both

$= \{\ \} \cup \{3, 6\} = \{3, 6\}$

$E \cap F$ = The elements which are common in both E and F

$= \{6\} \cap \{3, 4, 5, 6\} = \{6\}$

$D \cap E$ = The elements which are common in both D and E

$= \{1, 2, 3\} \cap \{6\} = \phi$

$A - C$ = The elements which are in A but not in C

$= \{1, 2, 3, 4, 5, 6\} - \{3, 6\} = \{1, 2, 4, 5\}$

$D - E$ = The elements which are in D but not in E

$= \{1, 2, 3\} - \{6\} = \{1, 2, 3\}$

F' = The elements which are not in F

$= (S - F)$
$= \{1, 2, 3, 4, 5, 6\} - \{3, 4, 5, 6\} = \{1, 2\}$

and $E \cap F' = E \cap (S - F)$

$= E \cap (\{1, 2, 3, 4, 5, 6\} - \{3, 4, 5, 6\})$
$= \{6\} \cap \{1, 2\} = \phi$

Mutually Exclusive Events

Two events are said to be mutually exclusive, if the occurrence of any one of them excludes the occurrence of other event, i.e. if they cannot occur simultaneously.

Thus, two events E_1 and E_2 are said to be mutually exclusive, if

$$E_1 \cap E_2 = \phi$$

e.g. In throwing a die, we have sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let E_1 = Event of getting even numbers $= \{2, 4, 6\}$

E_2 = Event of getting odd numbers $= \{1, 3, 5\}$

Here, $E_1 \cap E_2 = \phi$.

So, E_1 and E_2 are mutually exclusive events.

In general, events $E_1, E_2, \ldots, E_n$ are said to be mutually exclusive, if they are pairwise disjoint,

i.e. if $E_i \cap E_j = \phi$ for all $i \neq j$.

Note

Simple events of a sample space are always mutually exclusive.

EXAMPLE |7| An experiment involves rolling a pair of dice and recording the numbers that comes up. Describe the following events.

A : the sum is greater than 8.

B : 2 occurs on either die.

C : the sum is at least 7 and multiple of 3.

Which pairs of these events are mutually exclusive? **[NCERT]**

Sol. There are 36 elements in the sample space

$$S = \{(x, y): x, y = 1, 2, 3, 4, 5, 6\}$$

Given, A = events of getting the sum greater than 8, i.e. events of getting the sum 9, 10, 11 and 12.

$\therefore$ $A = \{(6,3), (3,6), (4,5), (5,4), (6,4), (4,6)$
$(5,5), (6,5), (5,6), (6,6)\}$

and B = event of getting 2 on either die.

$\therefore$ $B = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (1,2),$
$(3,2), (4,2), (5,2), (6,2)\}$

and C = event of getting the sum atleast 7 and a multiple of 3, i.e. event of getting the sum 9 and 12.

$= \{(4,5), (5,4), (6,3), (3,6), (6,6)\}$

Here, $A \cap B = \phi$, $B \cap C = \phi$

and $A \cap C \neq \phi$

Thus, we can say that, the pairs A and B; B and C are mutually exclusive events.

EXAMPLE |8| Three coins are tossed once. Let A denotes the event "three heads show", B denotes the event "two heads and one tail show", C denotes the event "three tails show" and D denotes the event "a head shows on the first coin". Which pair of events are **[NCERT]**

(i) mutually exclusive events. (ii) simple events.

(iii) compound events.

Sol. When three coins are tossed, then the sample space is

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Given, A = event of getting three heads

$\therefore$ $A = \{HHH\}$

B = event of getting two heads and one tail

$\therefore$ $B = \{HHT, HTH, THH\}$

C = event of getting three tails

$\therefore$ $C = \{TTT\}$

and D = event of getting a head on first coin.

$\therefore$ $D = \{HHH, HTT, HHT, HTH\}$

(i) Clearly, $A \cap B = \phi$, $A \cap C = \phi$,
$A \cap D = \{HHH\}$, $B \cap C = \phi$,
$B \cap D = \{HHT, HTH\}$ and $C \cap D = \phi$
$\therefore$ We can say. A and B; A and C; B and C; C and D are mutually exclusive events.

(ii) A and C are simple events, since both have only one sample point.

(iii) B and D are compound event, since they have more than one sample point.

Exhaustive Events

For a random experiment, a set of events is said to be exhaustive, if one of them necessarily occurs whenever the experiment is performed. Let $E_1, E_2, E_3, \ldots, E_n$ be n subsets of a sample space S. Then, events $E_1, E_2, E_3, \ldots, E_n$ are called exhaustive events, if

$$E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$$

e.g. Consider the experiment of throwing a die. We have, $S = \{1, 2, 3, 4, 5, 6\}$. Let us define the following events

E_1 : a number less than 4 appears.

E_2 : a number greater than 2 but less than 5 appears.

E_3 : a number greater than 4 appears.

Then, $E_1 = \{1, 2, 3\}$, $E_2 = \{3, 4\}$, $E_3 = \{5, 6\}$.

We observe that

$$E_1 \cup E_2 \cup E_3 = \{1, 2, 3\} \cup \{3, 4\} \cup \{5, 6\} = S$$

Thus, the events E_1, E_2 and E_3 are exhaustive events.

Note

If $E_i \cap E_j = \phi$ for $i \neq j$, i.e. events E_i and E_j are pairwise disjoint and $\bigcup_{i=1}^{n} E_i = S$, then events $E_1, E_2, \ldots, E_n$ are called mutually exclusive and exhaustive events.

EXAMPLE |9| A coin is tossed three times, consider the following events?

A : No head appears.

B : Exactly one head appears.

and C : Atleast two heads appears.

Do they form a set of mutually exclusive and exhaustive events? **[NCERT]**

Sol. When a coin is tossed three times, then the sample space is

$S = \{HHH, HHT, HTH, THH, THT, TTH, HTT, TTT\}$

Given, A = event of getting no head

$\therefore \quad A = \{TTT\}$

B = event of getting exactly one head

$\therefore \quad B = \{TTH, HTT, THT\}$

and C = event of getting atleast two heads

$\therefore \quad C = \{HHT, HTH, THH, HHH\}$

Clearly, $A \cap B = \phi$, $A \cap C = \phi$ and $B \cap C = \phi$

So, we can say that A, B and C are mutually exclusive events. [$\because$ A, B and C are pairwise disjoint]

Also, $A \cup B \cup C = \{TTT, TTH, HTT, THT, HHT, HTH, THH, HHH\} = S$

$\therefore$ A, B and C are exhaustive events.

Hence, A, B and C form a set of mutually exclusive and exhaustive events.

TOPIC PRACTICE 1

OBJECTIVE TYPE QUESTIONS

1 The total number of elementary events associated to the random experiment of throwing three die together is

(a) 210 (b) 216
(c) 215 (d) 220

2 If an event has more than one sample point, then it is called a/an

(a) simple event
(b) elementary event
(c) compound event
(d) None of the above

3 When the sets A and B are two events associated with a sample space. Then, event '$A \cup B$' denotes

(a) A and B (b) only A
(c) A or B (d) only B

4 The set $A - B$ denotes the event

(a) A and B (b) A or B
(c) only A (d) A but not B

5 A coin is tossed three times, consider the following events

A : no head appears.

B : exactly one head appears.

C : atleast two heads appear.

Then,

(a) A, B and C are mutually exclusive events
(b) A, B and C are exhaustive events
(c) Only (a)
(d) Both (a) and (b)

SHORT ANSWER Type I Questions

6 A coin is tossed. Find the total number of elementary events and also the total number of events associated with the random experiment.

7 Consider the experiment of rolling a die. Let A be the event 'getting a prime number' and B be the event 'getting an odd number'. Write the sets representing events **[NCERT]**

(i) A and B. (ii) A or B.
(iii) A but not B. (iv) not A.

8 Two dice are rolled. Let E_1, E_2 and E_3 be the events of getting a sum of 4, 5 and 6 respectively.

(i) Which events are elementary events?
(ii) Which events are compound events?

9 An experiment involves tossing of two coins and recording them in the following events

A : No tail
B : exactly one tail
C : at least one tail.

Write the sets representing events

(i) A and C. (ii) A not B.

10 A die is tossed. Let E denotes the event of getting a number multiple of 2 and F denotes the event of getting a number less than 7. Simplify the following event

(i) $E \cup F$ (ii) $F - E$
(iii) E' (iv) $E \cap F$

SHORT ANSWER Type II Question

11 A coin is tossed two times, consider the following events?

E_1 : No head appears.

E_2 : Exactly one head appears.

and E_3 : At least one head appears.

Do, they form a set of mutually exclusive and exhaustive events?

12 Two dice are rolled, A is the event that the sum of the numbers shown on the two dice is 5 and B is the event that atleast one of the dice shown up a 3. Are the two events A and B

(i) mutually exclusive? (ii) exhaustive?

13 Three coins are tossed once. Let A denotes the event "three tails show", B denotes the event "one tail and two head show", C denotes the event "tail on the first coin".

Which pair of events are

(i) mutually exclusive?

(ii) simple events?

(iii) compound events?

LONG ANSWER Type Questions

14 Two dice are thrown. The events A, B and C are as follows:

A : getting an even number on the first die.

B : getting an odd number on the first die.

C : getting the sum of the numbers on the dice ≤ 5.

Decribe the events

(i) A' (ii) not B

(iii) A or B (iv) A and B

(v) A but not C (vi) B or C

(vii) B and C (viii) $A \cap B' \cap C'$

15 Two dice are thrown and the sum of the numbers which come up on the dice is noted. Let us consider the following events associated with this experiment.

A : 'the sum is even'.

B : 'the sum is a multiple of 3'.

C : 'the sum is less than 4'.

D : 'the sum is greater than 11'.

Which pairs of these events are mutually exclusive? [NCERT]

HINTS & ANSWERS

1. (b) When three dice tossed together, then the total number of possible outcomes $= 6^3 = 6 \times 6 \times 6 = 216$

2. (c) If an event has more than one sample point, then it is called a compound event.

3. (c) When the sets A and B are two events associated with a sample space, then '$A \cup B$' is the event 'either A or B or both'. This event '$A \cup B$' is also called 'A or B'.
Therefore, event 'A or B' $= A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$

4. (d) We know that, $A - B$ is the set of all those elements which are in A but not in B. Therefore, the set $A - B$ may denote the event 'A but not B'.

5. (d) The sample space of the experiment is
$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$
$$A = \{TTT\},$$
$$B = \{HTT, THT, TTH\},$$
and $C = \{HHT, HTH, THH, HHH\}$
Now, $A \cup B \cup C = \{TTT, HTT, THT, TTH, HHT, HTH, THH, HHH\}$
$= S$
Therefore, A, B and C are exhaustive events.
Also, $A \cap B = \phi$, $A \cap C = \phi$ and $B \cap C = \phi$
Therefore, they are mutually exclusive.
Hence, A, B and C form a set of mutually exclusive and exhaustive events.

6. 2, 4

7. $A = \{2, 3, 5\}$ and $B = \{1, 3, 5\}$
Ans. (i) $\{3, 5\}$ (ii) $\{1, 2, 3, 5\}$ (iii) $\{2\}$ (iv) $\{1, 4, 6\}$

8. $E_1 = \{(1, 3), (2, 2), (3, 1)\}$
$E_2 = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$
$E_3 = \{(1, 5), (2, 4), (3, 3), (5, 1), (4, 2)\}$
Solve as Example 4.
Ans. (i) No event.
(ii) E_1, E_2 and E_3

9. $A = \{(H, H)\}$, $B = \{(H, T), (T, H)\}$
and $C = \{(H, T), (T, H), (T, T)\}$
(i) A and $C = A \cap C = \phi$
(ii) A not $B = A - B = \{(H, H)\}$

10. $E = \{2, 4, 6\}$ and $F = \{1, 2, 3, 4, 5, 6\}$
Ans. (i) $\{1, 2, 3, 4, 5, 6\}$ (ii) $\{1, 3, 5\}$
(iii) $\{1, 3, 5\}$ (iv) $\{2, 4, 6\}$

11. $E_1 = \{(T, T)\}$, $E_2 = \{(H, T), (T, H)\}$
and $E_3 = \{(H, T), (T, H), (H, H)\}$
Here, $E_1 \cap E_2 = \phi$, $E_1 \cap E_3 = \phi$ and $E_2 \cap E_3 \ne \phi$
Also, $E_1 \cup E_2 \cup E_3 = \{(H, T), (T, H), (T, T), (H, H)\} = S$
Ans. E_1, E_2 and E_3 are not mutually exclusive but they are exhaustive events.

12. (i) $A = \{(1, 4), (4, 1), (3, 2), (2, 3)\}$

and $B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (1, 3)$
$(2, 3), (4, 3), (5, 3), (6, 3)\}$

$\therefore \quad A \cap B = \{(3, 2), (2, 3)\} \neq \phi$

(ii) $A \cup B \neq S$

Ans. (i) No (ii) No

13. Solve as Example 8.

Ans. (i) A and B, B and C

(ii) A (iii) B and C.

14. (i) $A = \left\{\begin{matrix}(2,1),(2,2),\ldots,(2,6),\\(4,1),(4,2),\ldots,(4,6),\\(6,1),(6,2),\ldots,(6,6)\end{matrix}\right\}$, $B = \left\{\begin{matrix}(1,1),(1,2),\ldots,(1,6),\\(3,1),(3,2),\ldots,(3,6),\\(5,1),(5,2),\ldots,(5,6)\end{matrix}\right\}$

and $C = \{(1,1),(1,2),(2,1),(1,3),(3,1),$
$(2,2),(2,3),(3,2),(1,4),(4,1)\}$

Ans. (i) $A' = \left\{\begin{matrix}(1,1),(1,2),\ldots,(1,6),\\(3,1),(3,2),\ldots,(3,6),\\(5,1),(5,2),\ldots,(5,6)\end{matrix}\right\} = B$

(ii) $B' = \{(2,1),(2,2),(2,3),(2,4),(2,5),$
$(2,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\} = A$

(iii) $A \cup B = \{(1,1),(1,2),(1,3),(1,4),(1,5),$
$(1,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(2,1),$
$(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3),$
$(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\} = S$

(iv) $A \cap B = \phi$

(v) $A - C = \{(2,4),(2,5),(2,6),(4,2),(4,3),$
$(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

(vi) $B \cup C = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),$
$(3,4),(3,5),(3,6),(4,1),(5,1),(5,2),$
$(5,3),(5,4),(5,5),(5,6)\}$

(vii) $B \cap C = \{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(viii) $A \cap B' \cap C' = \{(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),$
$(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)$

15. Solve as Example 7. **Ans.** C and D

|TOPIC 2|

Axiomatic Approach to Probability

PROBABILITY OF OCCURRENCE OF AN EVENT

A numerical value that conveys the chance of occurrence of an event, when we perform an experiment, is called the probability of that event.

(i) Axiomatic approach to probability

Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval [0, 1] satisfying the following **axioms**

(i) For any event E, $P(E) \geq 0$

(ii) $P(S) = 1$

(iii) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

Now, let S be a sample space containing outcomes $E_1, E_2, \ldots, E_n$ i.e. $S = \{E_1, E_2, \ldots, E_n\}$.

Then, from the axiomatic definition of probability we have

- $0 \leq P(E_i) \leq 1, \forall E_i \in S$
- $P(E_1) + P(E_2) + \ldots + P(E_n) = 1$
- For any event A, $P(A) = \sum_{i=1}^{n} P(E_i)$, $E_i \in A$

Note (i) $P(\phi) = 0$.

(ii) For notational convenience, we write $P(E_i)$ for $P(\{E_i\})$.

EXAMPLE |1| Which of the following cannot be valid assignments of probabilities for outcomes of sample space. $S = \{W_1, W_2, W_3, W_4, W_5, W_6, W_7\}$?

Assignment	W_1	W_2	W_3	W_4	W_5	W_6	W_7
(i)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(ii)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(iii)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(iv)	–0.1	0.2	0.3	0.4	–0.2	0.1	0.3
(v)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

According to axiomatic approach to probability, we should have

(i) $0 \leq P(W_i) \leq 1 \,\forall\, W_i \in S$.

(ii) $P(W_1) + P(W_2) + \ldots + P(W_7) = 1$.

Sol. (i) Here, each of $P(W_i)$, where $i = 1$ to 7 is positive and less than 1.

Also, sum of probabilities
$= 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1$

$\therefore$ This assignment is valid.

(ii) Here, each of $P(W_i)$, where $i = 1$ to 7 is positive and less than 1. Also, sum of probabilities $= 7 \times \frac{1}{7} = 1$

$\therefore$ This assignment is also valid.

(iii) Here, each of $P(W_i)$, where $i = 1$ to 7 is positive and less than 1.

But sum of probabilities

$= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7$

$= 2.8 \neq 1$

$\therefore$ This assignment is not valid.

(iv) Here, $P(W_1)$ and $P(W_5)$ are negative.

$\therefore$ This assignment is not valid. $[\because 0 \le P(W_i) \le 1 \ \forall \ W_i \in S]$

(v) Here, $P(W_7) > 1$.

$\therefore$ This assignment is not valid. $[\because 0 \le P(W_i) \le 1 \ \forall \ W_i \in S]$

Hence, assignments (iii), (iv) and (v) are not valid.

(ii) STATISTICAL APPROACH TO PROBABILITY

In statistical approach, probability of an event 'A' is defined as

$$P(A) = \frac{\text{Number of trials in which the event A happened}}{\text{Total number of trials}}$$

EXAMPLE |2| A die is thrown 500 times with frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given in the following table

Outcome	1	2	3	4	5	6
Frequency	110	115	100	50	45	80

Find the probability of getting outcome 3 ?

Sol. Here,

Total frequency = Total number of times a die is thrown

= 500

and number of observed frequency, A

= Number of times the outcome 3 is thrown = 100

$\therefore$ Probability of getting outcome 3,

$$P(A) = \frac{\text{Number of observed frequency}}{\text{Total frequency}} = \frac{100}{500} = \frac{1}{5}$$

(iii) CLASSICAL APPROACH TO PROBABILITY

To obtain the probability of an event, we find the ratio of the number of outcomes favourable to the event, to the total number of equally likely outcomes. This theory is known as classical theory of probability.

i.e. $$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

This is also known as theoretical probability.

EXAMPLE |3| Suppose we throw a die once. Find the probability of getting a number greater than 4.

Sol. Let A be the event of getting a number greater than 4

Here, total number of possible outcomes

= Total number of outcomes on throwing a die = 6

and number of favourable outcomes

= Number of outcomes which are favourable to event A

$= \{5, 6\} = 2$

$\therefore$ Probability of getting a number greater than 4, $P(A) = \frac{2}{6}$

PROBABILITY OF EQUALLY LIKELY OUTCOMES

The outcomes of a random experiment are said to be **equally likely**, if the chance of occurrence of each outcome is same, i.e. chances of each outcome is same.

Let the sample space of an experiment is

$$S = \{s_1, s_2, \ldots, s_n\}$$

Also, let all the outcomes are equally likely.

i.e. $P(s_i) = p$, for all $s_i \in S$, where $0 \le p \le 1$

Since, by axiomatic approach to probability, $\sum_{i=1}^{n} P(s_i) = 1$

$\therefore \quad \underbrace{p + p + \cdots + p}_{(n \text{ times})} = 1$

$\Rightarrow \quad np = 1 \Rightarrow p = \frac{1}{n}$

Thus, $\quad P(s_i) = \frac{1}{n} \ \forall \ i = 1, 2, \ldots, n.$

Note

Let E be an event associated with a sample space S, such that $n(S) = n$ and $n(E) = m$, then by axiomatic approach

$$P(E) = \underbrace{\frac{1}{n} + \frac{1}{n} + \frac{1}{n} + \ldots + \frac{1}{n}}_{m \text{ times}} = m \times \frac{1}{n} = \frac{m}{n}$$

and by classical approach,

$$P(E) = \frac{\text{Number of favourable outcomes to } E}{\text{Total number of possible outcomes}} = \frac{m}{n}$$

Thus, in case of equally likely outcomes axiomatic approach coincide with the classical approach.

EXAMPLE |4| Suppose that each child born is equally likely to be a boy or a girl. Consider a family with exactly three children.

(i) List the eight elements in the sample space whose outcomes are all possible genders of the three children.

(ii) Write each of the following events as a set and find its probability.

(a) The event that exactly one child is a girl.

(b) The event that atleast two children are girls.

(c) The event that no child is a girl. **[NCERT Exemplar]**

Sol. (i) Let B denotes a boy and G denotes a girl. Then, all possible genders are expressed as

$$S = \{BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG\}$$

(ii) (a) Let E_1 denotes the event that exactly one child is a girl.

Then, $E_1 = \{BBG, BGB, GBB\}$

$\Rightarrow \quad P(E_1) = \frac{3}{8}$

(b) Let E_2 denotes the event that atleast two children are girls.

Then, $E_2 = \{BGG, GBG, GGB, GGG\}$

$\Rightarrow \quad P(E_2) = \frac{4}{8} = \frac{1}{2}$

(c) Let E_3 denotes the event that no child is a girl.

Then, $E_3 = \{BBB\} \Rightarrow P(E_3) = \frac{1}{8}$

EXAMPLE |5| One urn contains two black balls (labelled B_1 and B_2) and one white (W) ball. A second urn contains one black (B) ball and two white balls (labelled W_1 and W_2). Suppose the following experiment is performed.
One of the two urns is chosen at random. Next, a ball is randomly chosen from the urn. Then, a second ball is chosen at random from the same urn without replacing the first ball. What is the probability that two balls of opposite colour are chosen? **[NCERT Exemplar]**

Sol. Clearly, the sample space of the experiment is

$$S = \{B_1B_2, B_1W, B_2B_1, B_2W, WB_1, WB_2, BW_1, BW_2, W_1B, W_1W_2, W_2B, W_2W_1\}$$

Let E be the event that balls of opposite colour are chosen, then

$$E = \{B_1W, WB_1, B_2W, WB_2, BW_1, BW_2, W_1B, W_2B\}$$

$\therefore \quad P(E) = \frac{8}{12} = \frac{2}{3}$

Note In one-by-one selection, order of things is considered.

EXAMPLE |6| What is the probability that in a leap year chosen at random will contain 53 Sundays or 53 Mondays?

Sol. Since, a leap year has 366 days, so there are 52 weeks and 2 days. These 2 days can be {Sun-Mon, Mon-Tues, Tues-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun}.

$\therefore$ P(53 Sunday or 52 Monday)

$= P$(getting Sun-Mon, Mon-Tues, Sat-Sun) $= \frac{3}{7}$.

KNOWLEDGE PLUS

Odds of an Event

If an event E occurs in m ways and not occur in n ways, then we say that

(i) odds in favour of event $E = \frac{m}{n}$ or $m : n$

and corresponding probability $= \frac{m}{m+n}$

(ii) odds against the event $E = \frac{n}{m}$ or $n : m$

and corresponding probability $= \frac{n}{m+n}$

EXAMPLE |7| The odds in favour of an event are 2:4. Find the probability of the occurrence of this event.

Sol. We know that, if odds in favour of an event are m:n, then probability of occurrence of this event is $\frac{m}{m+n}$

$\therefore$ Required probability $= \frac{2}{2+4} = \frac{2}{6} = \frac{1}{3}$

EXAMPLE |8| If the odds against the occurrence of an event are 4:7, find the probability of occurrence of the event.

Sol. We know that, if odds against of an event are n:m, then probability of occurrence of this event is $\frac{m}{m+n}$.

$\therefore$ Required probability $= \frac{7}{7+4} = \frac{7}{11}$

Different Problems Based on Probability

|TYPE I|

PROBLEMS BASED ON COINS

EXAMPLE |9| A fair coin with 1 marked on one face and 6 marked on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12.

Sol. Let S be a sample space associated with the given experiment.

Then, $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$\Rightarrow n(S) = 12$

(i) Let A be the event of getting the sum of numbers 3.

Then, $A = \{(1, 2)\}$

$\Rightarrow \quad n(A) = 1$

$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{1}{12}$

(ii) Let B be the event of getting the sum of numbers 12.

Then, $B = \{(6, 6)\}$

$\Rightarrow \quad n(B) = 1$

$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{1}{12}$

EXAMPLE |10| Three coins are tossed once. Find the probability of getting

(i) 3 heads (ii) atleast 2 heads

(iii) atmost 2 heads (iv) exactly two tails

(v) no tail. **[NCERT]**

Sol. In random experiment of tossing three coins, the sample space is

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$\Rightarrow \quad n(S) = 8$

(i) Let E_1 be the event of getting 3 heads. Then, outcomes favourable to E_1 is $\{HHH\}$.

Thus, $n(E_1) = 1$,

$\therefore \quad P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{1}{8}$

(ii) Let E_2 be the event of getting atleast 2 heads.

Then, outcomes favourable to E_2 are $\{HHT, HTH, THH$ and $HHH\}$.

Thus, $n(E_2) = 4$

$\therefore \quad P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{4}{8} = \dfrac{1}{2}$

(iii) Let E_3 be the event of getting atmost 2 tails. Then, outcomes favourable to E_3 are $\{HHH, HHT, HTH, THH, TTH, THT, HTT\}$.

Thus, $n(E_3) = 7$

$\therefore \quad P(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{7}{8}$

(iv) Let E_4 be the event of getting exactly two tails.

Then, outcomes favourable to E_4 are $\{TTH, HHT, THT\}$.

Thus, $n(E_4) = 3$

$\therefore \quad P(E_4) = \dfrac{n(E_4)}{n(S)} = \dfrac{3}{8}$

(v) Let E_5 be the event of getting no tail. Then, outcomes favourable to E_5 is $\{HHH\}$.

Thus, $n(E_5) = 1$

$\therefore \quad P(E_5) = \dfrac{n(E_5)}{n(S)} = \dfrac{1}{8}$

|TYPE II|

PROBLEMS BASED ON DICE

EXAMPLE |11| A die is thrown. Find **[NCERT]**

(i) P (a prime number) (ii) P(a number ≥ 3)

(iii) P(a number ≤ 1) (iv) P(a number more than 6)

(v) P(a number less than 6)

Sol. In throwing a die, the sample space will be

$S = \{1, 2, 3, 4, 5, 6\}$

$\therefore \quad n(S) = 6$

(i) Let E_1 be the event of getting a prime number.

Then, $E_1 = \{2, 3, 5\}$

$\therefore \quad n(E_1) = 3$

Now, probability of getting a prime number,

$P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}$

(ii) Let E_2 be the event of getting a number ≥ 3.

Then, $E_2 = \{3, 4, 5, 6\}$

$\therefore \quad n(E_2) = 4$

Now, probability of getting a number ≥ 3,

$P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{4}{6} = \dfrac{2}{3}$

(iii) Let E_3 be the event of getting a number ≤ 1.

Then, $E_3 = \{1\}$

$\therefore \quad n(E_3) = 1$

Now, probability of getting a number ≤ 1,

$P(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{1}{6}$

(iv) Let E_4 be the event of getting a number more than 6.

Then, $E_4 = \{\} = \phi$

$\Rightarrow E_4$ is an impossible event.

$\therefore \quad n(E_4) = 0$

Now, probability of getting a number more than 6,

$P(E_4) = \dfrac{n(E_4)}{n(S)} = \dfrac{0}{6} = 0$

(v) Let E_5 be the event of getting a number less than 6.

Then, $E_5 = \{1, 2, 3, 4, 5\}$

$\therefore \quad n(E_5) = 5$

Now, probability of getting a number less than 6.

$P(E_5) = \dfrac{n(E_5)}{n(S)} = \dfrac{5}{6}$

EXAMPLE |12| Three dice are rolling simultaneously. Find the probability of getting same numbers on all dice.

Sol. In rolling of three dice, the number of possible outcomes are $n(S) = 6 \times 6 \times 6 = 216$.

Let E be the event of getting same numbers on all dice.

Then, the outcomes favourable to E are (1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5) and (6, 6, 6).

$\Rightarrow \quad n(E) = 6$

Now, required probability, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{6}{216} = \dfrac{1}{36}$.

|TYPE III|

PROBLEMS BASED ON PLAYING CARDS

EXAMPLE |13| A card is drawn from a pack of 100 cards numbered 1 to 100. Find the probability of drawing a number which is a square.

Sol. Here, a card is drawn at random from a pack of 100 cards numbered 1 to 100.

$\therefore$ Total number of possible outcomes $n(S) = 100$

Now, let A be the event of getting a number which is a square. Then, the outcomes favourable of A are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

$\Rightarrow \quad A = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

$\Rightarrow \quad n(A) = 10$

Hence, required probability,

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{100} = \frac{1}{10}$$

EXAMPLE |14| A card is selected from a pack of 52 cards.

(i) How many points are there in the sample space?

(ii) Calculate the probability that the card is an ace of spades.

(iii) Calculate the probability that the card is

(a) an ace (b) black card. **[NCERT]**

Sol. (i) A card is drawn at random from a pack of 52 cards, therefore there are 52 sample points in the sample space.

(ii) There is only one ace of spades, therefore number of favourable outcomes is 1.

Hence, required probability is $\frac{1}{52}$.

(iii) (a) Since, there are four ace in a pack of 52 cards, therefore number of favourable outcomes are 4.

Hence, required probability is $\frac{4}{52}$ or $\frac{1}{13}$.

(b) Since, there are 26 black cards in a pack of 52 cards, therefore number of favourable outcomes are 26.

Hence, required probability is $\frac{26}{52}$ or $\frac{1}{2}$.

|TYPE IV|

PROBLEMS BASED ON LETTERS/NUMBERS

EXAMPLE |15| If a letter is chosen at random from the English alphabet, then find the probability that the letter is

(i) a vowel (ii) a consonant

Sol. There are 26 letters in English alphabet, out of which 5 are vowels and 21 are consonants.

So, total number of possible outcomes = 26

Let E be the event of getting a vowel and F be the event of getting a consonant

Then, $n(E)$ = Number of outcomes favourable to $E = 5$

and $n(F)$ = Number of outcomes favourable to $F = 21$

(i) Now, Probability of getting a vowel, $P(E) = \frac{5}{26}$

(ii) Probability of getting a consonant, $P(F) = \frac{21}{26}$

EXAMPLE |16| A single letter is selected at random from the word 'PROBABILITY', then find the probability that letter is a vowel. **[NCERT Exemplar]**

Sol. There are 11 letters in the word PROBABILITY.

Since, a letter is selected randomly, therefore the possible outcomes are P, R, O, B, A, I, L, T, Y.

But here, outcomes are not equally likely

as $\quad P(\text{letter P}) = \frac{1}{11}$; $P(\text{letter } R) = \frac{1}{11}$

$P(\text{letter O}) = \frac{1}{11}$; $P(\text{letter B}) = \frac{2}{11}$

[$\because$ B repeated twice in the word]

$P(\text{letter A}) = \frac{1}{11}$; $P(\text{letter I}) = \frac{2}{11}$

$P(\text{letter L}) = \frac{1}{11}$; $P(\text{letter T}) = \frac{1}{11}$

and $\quad P(\text{letter Y}) = \frac{1}{11}$

Now, let E be event of getting a vowel. Then outcomes favourable to E are A, I and O.

$\Rightarrow \quad E = \{A, I, O\}$

Now, $\quad P(E) = P(A) + P(I) + P(O)$

[by axiomatic approach to probability]

$$= \frac{1}{11} + \frac{2}{11} + \frac{1}{11} = \frac{4}{11}$$

EXAMPLE |17| What is the probability that a number selected from the number 1, 2,..., 25 is a prime number? You may assume that each of the 25 numbers is equally likely to be selected.

Sol. Here, a number is selected at random from the numbers 1, 2, 3,..., 25

$\therefore$ Total number of possible outcomes, $n(S) = 25$

Now, let A be event of getting a prime number.

Then, the outcomes favourable to A are 2, 3, 5, 7, 11, 13, 17, 19, 23

$\Rightarrow A = \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \Rightarrow n(A) = 9$

Hence, required probability, $P(A) = \frac{n(A)}{n(S)} = \frac{9}{25}$

|TYPE V|

PROBLEMS BASED ON COMBINATIONS

When two or more things are selected, then to find the total outcomes or outcomes favourable to an event, we use the combination. Number of combinations out of n things taken r at a time is denoted by ^nC_r and it is defined as

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Note $^nC_r = {}^nC_{n-r}$

EXAMPLE |18| An urn contains 7 white, 5 black and 3 red balls. Two balls are drawn at random. Find the probability that

(i) both the balls are red.
(ii) one ball is red, other is black.
(iii) one ball is white.

Sol. Number of white balls = 7

Number of black balls = 5

Number of red balls = 3

Total number of balls = 7 + 5 + 3 = 15

Total number of ways of drawing 2 balls out of 15 balls

$$= {}^{15}C_2$$

$\Rightarrow$ Total number of possible outcomes = ${}^{15}C_{12}$

(i) If both balls are red, then number of favourable outcomes = ${}^{3}C_2$

Hence, P (drawing both balls are red)

$$= \frac{{}^{3}C_2}{{}^{15}C_2} = \frac{3}{\frac{15 \times 14}{2 \times 1}} = \frac{1}{35}$$

(ii) If one ball is red and other is black, then number of favourable outcomes = ${}^{3}C_1 \times {}^{5}C_1 = 3 \times 5 = 15$

Hence, P (drawing one red ball and one black ball)

$$= \frac{{}^{3}C_1 \times {}^{5}C_1}{{}^{15}C_2} = \frac{15}{\frac{15 \times 14}{2 \times 1}} = \frac{1}{7}$$

(iii) If one ball is white, then other ball can be red or black.

So, number of favourable outcomes = ${}^{7}C_1 \times {}^{8}C_1$

$$= 7 \times 8 = 56$$

Hence, P (drawing one white ball)

$$= \frac{56}{{}^{15}C_2} = \frac{56}{\frac{15 \times 14}{2 \times 1}} = \frac{8}{15}$$

EXAMPLE |19| While shuffling a pack of 52 playing cards, 2 are accidentally dropped. Find the probability that the missing cards to be of different colours.

[NCERT Exemplar]

Sol. Probability that the missing cards to be different colours

$$= \frac{{}^{26}C_1 \times {}^{26}C_1}{{}^{52}C_2}$$

[$\because$ in a pack of cards, there are two colours, each of them contain 26 cards]

$$= \frac{26 \times 26}{\frac{52 \times 51}{2 \times 1}} = \frac{26}{51}$$

EXAMPLE |20| Four cards are drawn at random from pack of 52 playing cards. Find the probability of getting

(i) all face cards
(ii) two red cards and two black cards
(iii) one card from each suit.

Sol. Total number of possible outcomes = ${}^{52}C_4$

(i) We know that, there are 12 face cards.

$\therefore$ Number of favourable outcomes = ${}^{12}C_4$

Hence, P (getting all face cards) = $\frac{{}^{12}C_4}{{}^{52}C_4}$

(ii) We know that, there are 26 red cards and 26 black cards.

$\therefore$ Number of favourable outcomes = ${}^{26}C_2 \times {}^{26}C_2$

Hence, P (getting 2 red and 2 black cards) = $\frac{({}^{26}C_2)^2}{{}^{52}C_4}$

(iii) We know that, there are 4 suits each having 13 cards.

$\therefore$ Number of favourable outcomes

$$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = (13)^4$$

Hence, P (getting one card from each suit) = $\frac{(13)^4}{{}^{52}C_4}$.

EXAMPLE |21| A bag contains tickets numbered from 1 to 20. Two tickets are drawn. Find the probability that both the tickets have prime numbers on them.

Sol. Out of 20 tickets, 2 tickets can be drawn in ${}^{20}C_2$ ways.

$\therefore$ Total number of possible outcomes = ${}^{20}C_2$

There are 8 prime numbers in 1 to 20, namely, 2, 3, 5, 7, 11, 13, 17 and 19.

Out of these 8 numbers 2 numbers can be selected in ${}^{8}C_2$ ways

$\therefore$ Number of favourable outcomes = ${}^{8}C_2$

Hence, required probability = $\frac{{}^{8}C_2}{{}^{20}C_2} = \frac{8 \times 7}{20 \times 19} = \frac{14}{95}$

EXAMPLE |22| In a lottery, a person chooses six different natural numbers at random from 1 to 20 and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game?

Sol. Total number of ways of selecting six different natural numbers out of first 20 natural numbers = ${}^{20}C_6$.

$\Rightarrow$ Total number of possible outcomes = ${}^{20}C_6$

Note that, number of favourable outcomes = 1

[$\because$ it is given that a persons win the prize if six selected numbers match with the six numbers already fixed by the lottery committee]

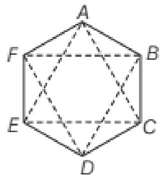
Hence, required probability = $\frac{1}{{}^{20}C_6} = \frac{1}{38760}$

EXAMPLE |23| Three of six vertices of a regular hexagon are chosen at random, what is the probability that the triangle with these vertices is equilateral?

[NCERT Exemplar]

Sol. Let $ABCDEF$ be a regular hexagon.

Clearly, out of 6 vertices, 3 vertices can be chosen in ${}^6C_3$ ways = 20 ways.



Thus, total number of triangles so formed = 20

[$\because$ no three points are collinear]

Note that, out of these 20 triangles only ΔACE and ΔBDF are equilateral triangles

$\therefore$ Required probability $= \frac{2}{20} = \frac{1}{10}$

|TYPE VI|

PROBLEMS BASED ON PERMUTATION OR ARRANGEMENTS

EXAMPLE |24| If the letters of the word ALGORITHM are arranged at random in a row, what is the probability the letters GOR must remain together as a unit? **[NCERT Exemplar]**

Sol. There are 9 letters in the word 'ALGORITHM'.

These 9 letters can be arranged in a row in 9! ways.

$\therefore$ Total number of possible outcomes = 9!

Now, if the letters GOR remain together, then we have 7 letters (considering GOR as 1 letter i.e. ALITHMGOR) which can be arranged in 7!.

Number of favourable outcomes = 7!

Hence, required probability $= \frac{7!}{9!} = \frac{1}{9 \times 8} = \frac{1}{72}$

EXAMPLE |25| Three letters are dictated to three persons and an envelope is addressed to each of them. The letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that atleast one letter is in its proper envelope. **[NCERT]**

Sol. Let L_1, L_2, L_3 be three letters and E_1, E_2, E_3 be their corresponding envelopes.

Clearly, 3 letters can be inserted into 3 addressed envelopes in ${}^3P_3 = 3! = 6$ ways, which are shown as follows

$\{(L_1,E_1),(L_2,E_3),(L_3,E_2);(L_1,E_2),(L_2,E_1),(L_3,E_3);$
$(L_1,E_3),(L_2,E_2),(L_3,E_1);(L_1,E_1),(L_2,E_2),(L_3,E_3);$
$(L_1,E_2),(L_2,E_3),(L_3,E_1);(L_1,E_3),(L_2,E_1),(L_3,E_2)\}$

Note that, there are 4 ways in which atleast one letter is in its proper envelope.

$\therefore$ Required probability $= \frac{4}{6} = \frac{2}{3}$

EXAMPLE |26| The letters of the word 'FORTUNATES' are arranged at random in a row. What is the chance that the two 'T' come together.

Sol. There are 10 letters in the word 'FORTUNATES' out of which 2 are T's.

$\therefore$ Total number of arrangements $= \frac{10!}{2!}$

$\Rightarrow$ Total number of possible outcomes $= \frac{10!}{2!}$

Now, if two 'T' come together, then we have 9 distinct letter (considering two 'T' as one letter) which can be arranged in 9! ways.

$\Rightarrow$ Number of favourable outcomes = 9!

Hence, required probability $= \frac{9!}{\left(\frac{10!}{2!}\right)} = \frac{9! \times 2}{10!} = \frac{2}{10} = \frac{1}{5}$

EXAMPLE |27| 6 boys and 6 girls sit in a row randomly. Find the probability that all 6 girls sit together. **[NCERT Exemplar]**

Sol. Total number of persons = 12, which can be arranged in a row in 12! ways.

Now, if 6 girls sit together, then we have 7 persons (considering 6 girls as one person), which can be in a row in 7! ways. But 6 girls can be arrange in 6! ways.

$\therefore$ Required probability

$$= \frac{7! \times 6!}{12!} = \frac{1}{132}$$

EXAMPLE |28| The number lock of a suitcase has four wheels, each labelled with 10 digits i.e. from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase? **[NCERT]**

Sol. When the digits are not repeated, then first place may have one of 10 digits, the second 9, third 8 and fourth 7.

Number of 4-digit numbers, $n(S) = 10 \times 9 \times 8 \times 7 = 5040$

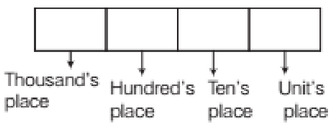
Now, lock can be opened only in 1 way.

$\therefore \quad n(E) = 1$

Hence, probability of opening the lock $= \frac{n(E)}{n(S)} = \frac{1}{5040}$

EXAMPLE |29| Without repetition of the number, four digit numbers are formed with the numbers 0, 2, 3, 5. Find the probability of such a numbers is divisible by 5. **[NCERT Exemplar]**

Sol. In a four digit number, 0 cannot put in thousand's place. So, the number of ways of filling up the thousand's place = 3.



Now, we left with three digits (including 0) and 3 places. Note that, hundred's place can be filled up in 3 ways. ten's place can be filled up in 2 ways and unit's place can be filled up in 1 ways

So, the number of 4-digits numbers $= 3 \times 3 \times 2 \times 1 = 18$

$\Rightarrow$ Total number of possible outcomes $= 18$

Now, if the number is divisible by 5, then its unit's place is filled by either 5 or 0.

***Case* I** When units place filled by 0.

			0

In this case, thousand's place can be filled up in 3 ways, hundred's place can be filled up in 2 ways and ten's place can be filled up in 1 ways.

Thus, in this case, number of four-digits number divisible by $5 = 3 \times 2 \times 1 = 6$

***Case* II** When unit's place is filled by 5.

			5

In this case, thousand's place can be filled up in 2 ways. hundred's place can be filled up in 2 ways and ten's place can be filled up in 1 ways

Thus, in this case, number of four-digits number divisible by $5 = 2 \times 2 \times 1 = 4$. From cases I and II,

Total number of 4-digits numbers divisible by 5

$$= 6 + 4 = 10$$

$\Rightarrow$ Number of favourable outcome $= 10$

Hence, required probability $= \frac{10}{18} = \frac{5}{9}$

EXAMPLE |30| On her vacations, Veena visits four cities (A, B, C and D) in a random order. What is the probability that she visits

(i) A before B?
(ii) A before B and B before C?
(iii) A first and B last?
(iv) A either first or second?
(v) A just before B? **[NCERT]**

Sol. Number of ways, in which Veena can visit four cities A, B, C and D, is 4! i.e. 24.

$$\therefore \quad n(S) = 24$$

Clearly, sample space for this experiment is

$$S = \begin{Bmatrix} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, \\ BACD, BADC, BCAD, BCDA, BDAC, BDCA, \\ CABD, CADB, CBAD, CBDA, CDAB, CDBA, \\ DABC, DACB, DCAB, DCBA, DBAC, DBCA \end{Bmatrix}$$

(i) Let E_1 be the event that Veena visits A before B. Then,

$$E_1 = \begin{Bmatrix} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, \\ CABD, CADB, CDAB, DABC, DACB, DCAB \end{Bmatrix}$$

$$\Rightarrow \quad n(E_1) = 12$$

$$\therefore P(\text{she visits } A \text{ before } B) = P(E_1) = \frac{n(E_1)}{n(S)} = \frac{12}{24} = \frac{1}{2}$$

(ii) Let E_2 be the event that she visits A before B and B before C.

Then $\quad E_2 = \{ABCD, ABDC, ADBC, DABC\}$

$\Rightarrow \quad n(E_2) = 4$

$\therefore P$ (she visits A before B and B before C) $= P(E_2)$

$$= \frac{n(E_2)}{n(S)} = \frac{4}{24} = \frac{1}{6}$$

(iii) Let E_3 be the event that she visits A first and B last.

Then, $E_3 = \{ACDB, ADCB\} \Rightarrow n(E_3) = 2$

$\therefore P$ (she visits A first and B last) $= P(E_3)$

$$= \frac{n(E_3)}{n(S)} = \frac{2}{24} = \frac{1}{12}$$

(iv) Let E_4 be the event that she visits A either first or second. Then,

$$E_4 = \begin{Bmatrix} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, \\ BACD, BADC, CABD, CADB, DABC, DACB \end{Bmatrix}$$

$\Rightarrow \quad n(E_4) = 12$

Hence, P (she visits A either first or second)

$$= P(E_4) = \frac{n(E_4)}{n(S)} = \frac{12}{24} = \frac{1}{2}$$

(v) Let E_5 be the event that she visits A just before B.

Then, $E_5 = \{ABCD, ABDC, CABD, CDAB, DABC, DCAB\}$

$\Rightarrow \quad n(E_5) = 6$

Hence, P (she visits A just before B)

$$= P(E_5) = \frac{n(E_5)}{n(S)} = \frac{6}{24} = \frac{1}{4}$$

|TYPE VII|

MISCELLANEOUS PROBLEMS

EXAMPLE |31| There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Sol. Total number of person = 4 (men) + 6 (women) = 10

Since, a person is selected at random, therefore total number of possible outcomes = 10

Now, let A be the event of selecting a woman. Then number of outcomes favourable to A are 6.

Hence, probability of selecting a woman, $P(A) = \frac{6}{10} = \frac{3}{5}$

EXAMPLE |32| A bag contains 9 red, 7 white and 4 black balls. A ball is drawn at random. Find the probability that the ball drawn is

(i) red
(ii) white
(iii) not black

Sol. Total number of balls = 9 red + 7 white + 4 black

$$= 20$$

$\Rightarrow$ Total number of possible outcomes, $n(S) = 20$

(i) Let E_1 be the event of getting a red ball. Then, $n(E_1) = 9$

Now, P (getting a red ball) $= P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{20}$

(ii) Let E_2 be the event of getting a white ball.

Then, $n(E_2) = 7$

Now, P (getting a white ball) $= P(E_2) = \frac{n(E_2)}{n(S)} = \frac{7}{20}$

(iii) Let E_3 be the event of getting a non-black ball. Then, $n(E_3) = 9 + 7 = 16$

Now, P (getting a non-black ball)

$$= P(E_3) = \frac{n(E_3)}{n(S)} = \frac{16}{20} = \frac{4}{5}$$

EXAMPLE |33| One mapping (function) is selected at random from all the mappings of the set $A = \{1, 2, 3, \dots, n\}$ into itself. Find the probability that the mapping selected is one to one.

Sol. We know that, total number of mappings from a set A having n elements into itself is n^n.

and the total number of one to one mapping is $n!$.

$\therefore$ Required probability $= \frac{n!}{n^n}$

Note

In one to one mapping, distinct element of a domain have distinct image.

TOPIC PRACTICE 2

OBJECTIVE TYPE QUESTIONS

1 Let S be a sample space containing outcomes $\omega_1, \omega_2, \omega_3, \dots, \omega_n$ *i.e.,* $S = \{\omega_1, \omega_2, \dots, \omega_n\}$.

Then, which of the following is true?

I. $0 \le P(\omega_i) \le 1$ for each $\omega_i \in S$

II. $P(\omega_1) + P(\omega_2) + \dots + P(\omega_n) = 1$

III. For any event A, $P(A) = \Sigma P(\omega_i)$, $\omega_i \in A$

(a) Only I
(b) Only II
(c) Only III
(d) All of the above

2 A coin is tossed twice. Then, the probability that atleast one tail occurs is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{3}{4}$

3 Three numbers are chosen from 1 to 20. Find the probability that they are not consecutive.

(a) $\frac{186}{190}$ (b) $\frac{187}{190}$ (c) $\frac{188}{190}$ (d) $\frac{18}{^{20}C_3}$

4 Three dice are thrown together. The probability of getting a total of at least 6 is

(a) $\frac{101}{108}$ (b) $\frac{103}{108}$ (c) $\frac{105}{108}$ (d) $\frac{107}{108}$

5 A card is drawn from an ordinary pack of 52 cards and a gambler bets that, it is a spade or an ace. The odds against his winning this bet is

(a) 4 : 9 (b) 9 : 4 (c) 3 : 8 (d) 8 : 3

VERY SHORT ANSWER Type Questions

6 The odds in favour of an event are 3 : 5. Find the probability of the occurrence of this event.

7 If the odds against the occurrence of an event are 3 : 5. Find the probability of occurrence of the event.

8 If E_1, E_2, E_3, E_4 are the four elementary outcomes in a sample space and $P(E_1) = 0.1$, $P(E_2) = 0.5$, $P(E_3) = 0.1$, then find the probability of E_4.

9 A bag contains 4 white and 5 black balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is white.

10 A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that the drawn balls are of different colours.

SHORT ANSWER Type I Questions

11 Let a sample space be $S = \{\omega_1, \omega_2, \dots, \omega_6\}$. Which of the following assignments of probabilities to each outcome are valid?

Outcomes	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
(i)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
(ii)	1	0	0	0	0	0
(iii)	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{3}$
(iv)	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{3}{2}$
(v)	0.1	0.2	0.3	0.4	0.5	0.6

12 A coin is tossed twice, what is the probability that at least one head occurs?

13 In a simultaneous toss of two coins, find the probability of
(i) exactly 1 tail (ii) no tail.

14 What is the probability of drawing a 'King' from a well-shuffled deck of 52 cards?

15 A bag contains 5 green and 7 red balls. Two balls are drawn. What is the probability that one is green and other is red?

16 Two cards are drawn at random from a pack of 52 cards. What is the probability that both the drawn cards are aces?

17 4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade? [NCERT]

18 A die is rolled. If the outcome is an odd number, then what is the probability that it is a prime number?

19 Find the probability that in a random arrangement of the letters of the word 'SOCIAL' vowels come together.

20 What is the probability that in a leap year chosen at random will contain 53 Tuesday or 53 Wednesday?

21 Two cards are drawn without replacement from a well-shuffled pack of 52 cards. What is the probability that one is red queen and the other is a king of black colour?

22 The letters of the word 'CLIFTON' are arranged at random in a row. What is the chance that two vowels come together?

23 What is the probability that a number selected from the number 1, 2, 3, ... , 35 is a prime number?

24 Find the probability that in a random arrangement of the letters of the word 'UNIVERSITY' the two I's come together.

25 There are 4 letters and 4 addressed envelopes. Find the probability that all the letters are not dispatched in right envelopes.

26 One mapping is selected at random from all the mappings of the set $B = \{1, 2, 3, \dots, 6\}$ into itself.

Find the probability that the mapping selected is one-to-one.

27 In a lottery, a person choose 4 different natural numbers at random from 1 to 30 and if there four numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game?

28 Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that this number has the same digits? [NCERT Exemplar]

29 If letters of the word PENCIL are arranged in random order, then what is the probability that N is always next to E?

SHORT ANSWER Type II Questions

30 A card is selected from a pack of 52 cards.
(i) How many points are there in the sample space.
(ii) Find the probability that the card is an ace of club.
(iii) Find the probability that the card is
(a) a king (b) red card

31 Three coins are tossed once. Find the probability of getting
(i) 2 heads (ii) no head
(iii) 3 tails (iv) atmost two tails

32 A die is thrown, find the probability of getting
(i) an even number.
(ii) a number less than or equal to 4.
(iii) a number less than or equal to 6.

33 Two dice are thrown simultaneously. Find the probability of getting
(i) an even number as the sum
(ii) a doublet
(iii) a doublet of even number
(iv) a multiple of 3 as the sum

34 From a well-shuffled deck of 52 cards, a card is drawn at random. Find the probability of getting
(i) a heart (ii) a red card
(iii) a face card (iv) an eight of hearts

35 A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is
(i) a vowel.
(ii) a consonant.

36 A single letter is selected at random from the word 'ASSESSMENT', then find the probability that letter is a vowel.

37 20 cards are numbered from 1 to 20. In those of them, one card is drawn at random. What is the probability that the number on the card drawn is

(i) a prime number? (ii) an odd number?
(iii) a multiple of 5? (iv) not divisible by 3?

38 A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that

(i) all the three balls are white.
(ii) all the three balls are red.
(iii) one ball is red and two balls are white.

39 A box contains 10 bulbs, of which just three are defective. If a random sample of five bulbs is drawn, find the probability that the sample contains

(i) all defective bulbs.
(ii) good bulbs.
(iii) exactly 2 defective bulbs.

40 A committee of two persons is selected from two men and two women. What is the probability that the committee will have

(i) no man?
(ii) one man?
(iii) two men?

41 A box contains 150 bulbs, 40 of which are defective. 20 bulbs are selected for inspection. Find the probability that

(i) no bulb is defective.
(ii) exactly one bulb is defective.
(iii) exactly two bulbs are defective.

42 A five digit number is formed by the digits 1,2,3,4,5 without repetition. Find the probability that the number is divisible by 4.

43 A combination lock on a suitcase has 3 wheels, each labelled with nine digits from 1 to 9. If an opening combination is a particular sequence of three digits with no repeats, what is the probability of a person guessing the right combination?

44 Three squares of chess board are selected at random. Find the probability of getting 2 squares of one colour and other of a different colour. [NCERT Exemplar]

45 In a relay race, there are five teams A, B, C, D and E.

(i) What is the probability that A, B and C finish first, second and third, respectively.
(ii) What is the probability that A, B and C are first three to finish (in any order), assume that all finishing orders are equally likely.

[NCERT]

46 A typical PIN (personal identification number) is a sequence of any four symbols chosen from the 26 letters in the alphabet and the ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol? [NCERT Exemplar]

47 What is the probability that in a group of 2 people, both will have the same birthday, assuming that there are 365 days in a year and no one has his/her birthday on 29th February.

48 Two balls are drawn at random from a bag containing 3 white, 3 red, 4 green and 4 black balls, one-by-one without replacement. Find the probability that both the balls are of different colour.

49 A bag contains 8 red, 3 white and 9 blackballs. If three balls are drawn at random, determine the probability that

(i) all the three balls are of blue colour.
(ii) all the balls are of different colours.
(iii) one is red and two are white.

50 A group consist of 3 men, 2 women and 4 children. If four persons are selected at random, find the probability of selecting

(i) 1 man, 1 woman and 2 children.
(ii) exactly 2 children.
(iii) 2 women.

51 A die is rolled in such a way that each odd number is twice as likely to occur as each even number. Find $P(G)$, where G is the event that a number greater than 3 occurs on a single roll of the die. [NCERT Exemplar]

LONG ANSWER Type Questions

52 In a lottery of 50 tickets numbered 1 to 50, two tickets are drawn simultaneously. Find the probability that

(i) both the tickets drawn have prime number.
(ii) none of the tickets drawn have prime number.
(iii) one ticket has prime number.

53 If 4-digit numbers greater than 5000 are randomly formed from the digits 0, 1, 3, 5 and 7. What is the probability of forming a number divisible by 5 when,

(i) the digits are repeated?

(ii) the repetition of digits are not allowed? **[NCERT]**

HINTS & ANSWERS

1. (d) Let S be a sample space containing outcomes $\omega_1, \omega_2, \ldots \omega_n$, i.e. $S = \{\omega_1, \omega_2, \ldots \omega_n\}$

It follows from the axiomatic definition of probability that

(i) $0 \le P(\omega_i) \le 1$ for each $\omega_i \in S$

(ii) $P(\omega_1) + P(\omega_2) + \ldots + P(\omega_n) = 1$

(iii) For any event A, $P(A) = \Sigma P(\omega_i)$, $\omega_i \in A$.

2. (d) The sample space is $S = \{HH, HT, TH, TT\}$

Let E be the event of getting atleast one tail

$E = \{HT, TH, TT\}$

$\therefore$ Required probability P

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{n(E)}{n(S)} = \frac{3}{4}$$

3. (b) Three numbers chosen from

$1 \text{ to } 20 = {}^{20}C_3 = \dfrac{20 \times 19 \times 18 \times 17!}{17! \times 3 \times 2 \times 1} = 20 \times 19 \times 3 = 1140$

Now set of three consecutive numbers = 18

$P(\text{they are consecutive}) = \dfrac{18}{1140} = \dfrac{3}{190}$

$P(\text{they are not consecutive}) = 1 - \dfrac{3}{190} = \dfrac{187}{190}$

4. (b) The total number of elementary events

$= 6 \times 6 \times 6 = 216.$

Let A be the event of getting a total of at least 6. Then, $\overline{A}$ denotes the event of getting a total of less than 6 i.e. 3, 4, 5.

$\therefore \quad \overline{A} = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1)\}$

So, $n(\overline{A}) = 10$

$\therefore \ P(\overline{A}) = \dfrac{10}{216}$

$\therefore \ P(A) = 1 - P(\overline{A}) = 1 - \dfrac{10}{216} = \dfrac{103}{108}$

5. (b) Let A be the event of getting a spade or an ace from a pack of 52 cards.

Then, total number of possible outcomes $= {}^{52}C_1 = 52$

Since, there are 13 spade cards including an ace of spade and three aces other than an ace of spade.

$\therefore$ Number of favourable outcomes $= {}^{16}C_1 = 16$

$\therefore \quad P(A) = \dfrac{16}{52} = \dfrac{4}{13}$

Hence, odds against A are $P(\overline{A}) : P(A) = \dfrac{9}{13} : \dfrac{4}{13} = 9:4$

6. Solve as Example 7. **Ans.** $\frac{3}{8}$

7. Solve as Example 8. **Ans.** $\frac{5}{8}$

8. By axiomatics approach to probability, we have $P(E_1) + P(E_2) + P(E_3) + P(E_4) = 1$ **Ans.** 0.3

9. Probability $= \dfrac{{}^4C_1}{{}^9C_1}$ **Ans.** $\frac{4}{9}$

10. Let E_2 be the event of getting all balls of different colours.

then $n(E_2) = {}^6C_1 \times {}^4C_1 \times {}^8C_1 = 192$

Required probability $= \dfrac{192}{{}^{18}C_3}$ **Ans.** $\frac{4}{17}$

11. Solve as Example 1. **Ans.** (i) and (ii)

12. $S = \{HH, HT, TH, TT\}$

$\therefore n(S) = 4$

n (at least one head occur) = 3

Required probability $= \dfrac{3}{4}$ **Ans.** $\frac{3}{4}$

13. (i) Favourable outcomes $= \{HT, TH\}$

(ii) Favourable outcome $= \{HH\}$ **Ans.** (i) $\frac{1}{2}$ (ii) $\frac{1}{4}$

14. Probability $= \dfrac{{}^4C_1}{{}^{52}C_1}$ **Ans.** $\frac{1}{13}$

15. Solve as Example 18 part (ii). **Ans.** $\frac{35}{36}$

16. Number of favourable outcomes ${}^4C_2$. **Ans.** $\frac{1}{221}$

17. Total number of possible outcomes $= {}^{52}C_4$

Number of favourable outcomes $= {}^{13}C_3 \times {}^{13}C_1$.

Ans. $\dfrac{{}^{13}C_3 \cdot {}^{13}C_1}{{}^{52}C_4}$

18. On rolling a die, we get outcomes as odd number. So, sample space, $S = \{1, 3, 5\} \Rightarrow n(S) = 3$

Let E be the event of getting a prime number then, $E = \{3, 5\} \Rightarrow n(E) = 2$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{2}{3}$

19. Total outcomes = 6!

Favourable outcomes $= 4! \times 3!$ **Ans.** $\frac{1}{5}$

20. Solve as Example 6. **Ans.** $\frac{3}{7}$

21. Number of ways of selecting 2 cards out of 52 cards

$= {}^{52}C_2 = 26 \times 51$

$\Rightarrow$ Total number of possible outcomes = 26×51

We know that, there are 2 red queen and 2 kings of black colour in a pack of 52 cards

$\therefore$ Number of favourable outcomes

$$= {}^2C_1 \times {}^2C_1 = 2 \times 2 = 4$$

Hence, required probability = $\frac{4}{26 \times 51} = \frac{2}{663}$

22. There are 7 letters in the word 'CLIFTON'.

These 7 letters can be arranged in a row in 7! ways.

$\Rightarrow$ Total number of possible outcomes = 7!

Now, if vowels (I, O) come together, then we have 6 letters (considering vowel as one letter i.e. CLFTN IO) which can be arranged in 6! ways.

But two vowels can be put together in 2! ways.

$\therefore$ The total number of arrangements in which two vowels come together is $6! \times 2!$

$\Rightarrow$ Number of favourable outcomes = $6! \times 2!$

Hence, required probability = $\frac{6! \times 2!}{7!} = \frac{2}{7}$

23. Prime numbers are {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31}

Ans. $\frac{11}{35}$

24. Solve as Example 26. **Ans.** $\frac{1}{5}$

25. Clearly, 4 letters can be inserted into 4 addressed envelope in ${}^4P_4 = 4! = 24$ ways.

$\Rightarrow$ Total number of possible ways is 24.

$\therefore$ P (all letters are not dispatched in right envelopes)

$= 1 - P$ (all letters are dispatched in right envelope)

$= 1 - \frac{1}{24}$ [$\because$ there is exactly one way in which all letters are dispatched in right envelope]

Ans. $\frac{23}{24}$

26. Solve as Example 33. **Ans.** $\frac{6!}{6^6}$

27. Solve as Example 22. **Ans.** $\frac{1}{{}^{30}C_4}$

28. Since, a 3-digit number cannot start with 0, therefore hundredth place can be filled up in 4 ways.

Now, as repetition of digits is allowed, therefore each of ten's and unit's place can be filled in 5 ways.

Thus, the total number of possible 3-digit number

$$= 4 \times 5 \times 5 = 100$$

Note that, there are three digits numbers, which has the same digits, *viz.* 222, 444, 666 and 888.

$\Rightarrow$ Number of favourable outcomes = 4

Hence, P (3-digit number with same digits) = $\frac{4}{100} = \frac{1}{25}$

29. There are 6 letters in the word 'PENCIL', which can be arranged in 6! ways.

$\Rightarrow$ Total number of possible outcomes = 6!

Now, if letter N is always next to E, then we have 5 letters (considering EN as one letter i.e. PCIL EN) which can be arranged in 5! ways

$\Rightarrow$ Number of favourable outcomes = 5!

Hence, required probability = $\frac{5!}{6!} = \frac{1}{6}$.

30. Solve as Example 14.

Ans. (i) 52 (ii) $\frac{1}{52}$ (iii) (a) $\frac{1}{13}$ (b) $\frac{1}{2}$

31. Solve as Example 10.

Ans. (i) $\frac{3}{8}$ (ii) $\frac{7}{8}$ (iii) $\frac{1}{8}$ (iv) $\frac{7}{8}$

32. Solve as Example 11.

Ans. (i) $\frac{1}{2}$ (ii) $\frac{2}{3}$ (iii) 1

33. Total number of outcomes = 36 $\Rightarrow n(s) = 36$

(i) Possible outcomes are (1, 1) (1, 3) (3, 1), (2, 2), (1, 5), (5, 1), (2, 4), (4, 2), (3, 3), (3, 5), (5, 3), (4, 4), (4, 6), (2, 6), (6, 2), (6, 4), (5, 5), (6, 6) $\therefore n(E) = 18$

(ii) Possible outcomes are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

(iii) Possible outcomes are (2, 2), (4, 4), (6, 6).

(iv) Possible outcomes are (1, 2), (2, 1), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6).

Ans.(i) $\frac{1}{2}$ (ii) $\frac{1}{6}$ (iii) $\frac{1}{12}$ (iv) $\frac{1}{3}$

34. (i) Required probability = $\frac{13}{52}$

(ii) Required probability = $\frac{26}{52}$

(iii) Required probability = $\frac{12}{52}$

(iv) Required probability = $\frac{1}{52}$

Ans. (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{3}{13}$ (iv) $\frac{1}{52}$

35. Solve as Example 15. **Ans.** (i) $\frac{6}{13}$ (ii) $\frac{7}{13}$

36. Solve as Example 16. **Ans.** $\frac{3}{10}$

37. (i) Favourable outcomes are 2, 3, 5, 7, 11, 13, 17 and 19.

Ans. $\frac{2}{5}$

(ii) Favourable outcomes are 1, 3, 5, 7, 9, 11, 13, 15, 17 and 19.

Ans. $\frac{1}{2}$

(iii) Favourable outcomes are 5, 10, 15 and 20. **Ans.** $\frac{1}{5}$

(iv) Favourable outcomes are 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20. **Ans.** $\frac{7}{10}$

38. (i) Number of favourable outcomes = ${}^5C_3$

Total number of possible outcomes = ${}^{13}C_3$ **Ans.** $\frac{5}{143}$

(ii) Number of favourable outcomes = ${}^8C_3$ **Ans.** $\frac{28}{143}$

(iii) Number of favourable outcomes= ${}^8C_1 \times {}^5C_2$ **Ans.** $\frac{40}{143}$

39. Total outcomes = ${}^{10}C_5$, defective bulbs = 3

Non-defective bulbs = 7

Ans. (i) $\frac{{}^3C_3 \times {}^7C_2}{{}^{10}C_5}$ (ii) $\frac{{}^7C_5}{{}^{10}C_5}$ (iii) $\frac{{}^3C_2 \times {}^7C_3}{{}^{10}C_5}$

40. Total number of possible outcomes = ${}^4C_2$

(i) Number of favourable outcomes = ${}^2C_2$ **Ans.** $\frac{1}{6}$

(ii) Number of favourable outcomes = ${}^2C_1 \times {}^2C_1$ **Ans.** $\frac{2}{3}$

(iii) Number of favourable outcomes = ${}^2C_2$ **Ans.** $\frac{1}{6}$

41. Total outcomes = ${}^{150}C_{20}$

Number of defective bulbs = 40

Number of non-defective bulbs = 110

Ans. (i) $\frac{{}^{110}C_{20}}{{}^{150}C_{20}}$ (ii) $\frac{{}^{40}C_1 \times {}^{110}C_{19}}{{}^{150}C_{20}}$ (iii) $\frac{{}^{40}C_2 \times {}^{110}C_{18}}{{}^{150}C_{20}}$

42. For a number to be divisible by 4, last two digits should be divisible by 4. (i.e. 12, 24, 32, 52).

Favourable cases = $3! \times 4$ **Ans.** $\frac{1}{5}$

43. Solve as Example 28. **Ans.** $\frac{1}{504}$

44. We know that, in a chess board, there are 64 squares of which 32 are white and 32 are black.

Clearly, 3 squares can be selected in ${}^{64}C_3$ ways.

Let E be the event of getting 2 squares of one colour and other of a different colour.

Then, $n(E) = {}^{32}C_2 \times {}^{32}C_1 + {}^{32}C_1 \times {}^{32}C_2$

[∵ 2 squares of one colour and 1 of other can be $2W$, $1B$ or $1W$, $2B$]

$= 2 \times {}^{32}C_2 \times {}^{32}C_1$ **Ans.** $\frac{16}{21}$

45. The sample space consisting of all finishing orders in the first three places, will have

${}^5P_3$, i.e. $\frac{5!}{(5-3)!} = 5 \times 4 \times 3 = 60$ sample points, each with a probability of $\frac{1}{60}$.

(i) If A, B and C finish first, second and third respectively, then there is only one finishing order for this, i.e. ABC.

Ans. $\frac{1}{60}$

(ii) If A, B and C are the first three finishers, then there will be 3! arrangements for A, B and C. Thus, the sample points corresponding to this event will be 3! in number.

Hence, P(A, B and C are first three to finish)

$= \frac{3!}{60}$ **Ans.** $= \frac{1}{10}$

46. Total number of symbols = 36

There are $36 \times 36 \times 36 \times 36 = (36)^4 = 1679616$ PINs in all.

[by fundamental principle of counting]

Note that, when repetition of symbols is not allowed, then there are $36 \times 35 \times 34 \times 33 = 1413720$ different PINs.

Now, the number of PINs that contain atleast one repeated symbol = 1679616 − 1413720 = 265896

Ans. 0.1583

47. Clearly, the total number of ways in which two persons may have their birthday

$= 365 \times 365 = (365)^2$

[∵ first person may have birthday in anyone of 365 days. Similarly, second person may have birthday in anyone of 365 days]

Note that, the number of ways in which two persons have the same birthday = 365 **Ans.** $\frac{1}{365}$

48. Total number of balls = 3 + 3 + 4 + 4 = 14

Since, two balls are to be drawn one-by-one without replacement.

There will be 4 cases

	First ball	Second ball	Probability
Case I	1 White (3)	1 Non-white (11)	$\frac{{}^3C_1}{{}^{14}C_1} \times \frac{{}^{11}C_1}{{}^{13}C_1} = \frac{3}{14} \times \frac{11}{13}$
Case II	1 Red (3)	1 Non-red (11)	$\frac{{}^3C_1}{{}^{14}C_1} \times \frac{{}^{11}C_1}{{}^{13}C_1} = \frac{3}{14} \times \frac{11}{13}$
Case III	1 Green (4)	1 Non-green (10)	$\frac{{}^4C_1}{{}^{14}C_1} \times \frac{{}^{10}C_1}{{}^{13}C_1} = \frac{4}{14} \times \frac{10}{13}$
Case IV	1 Black (4)	1 Non-black (10)	$\frac{{}^4C_1}{{}^{14}C_1} \times \frac{{}^{10}C_1}{{}^{13}C_1} = \frac{4}{14} \times \frac{10}{13}$

∴ Required probability

$$= \frac{3}{14} \times \frac{11}{13} + \frac{3}{14} \times \frac{11}{13} + \frac{4}{14} \times \frac{10}{13} + \frac{4}{14} \times \frac{10}{13}$$

Ans. $\frac{73}{91}$

49. Total number of balls $= 8 + 3 + 9 = 20$

Number of ways of selecting 3 balls out of 20 balls $= {}^{20}C_3$

$\Rightarrow$ Total number of possible outcomes $= {}^{20}C_3$

(i) Let E_1 be the event of getting all balls of blue colour.

Then, $n(E_1) = {}^9C_3$

Hence, required probability

$$= \frac{{}^9C_3}{{}^{20}C_3} = \frac{9\times8\times7}{20\times19\times18} = \frac{7}{95}$$

(ii) Let E_2 be the event of getting all balls of different colours. Then,

$$n(E_2) = {}^8C_1 \times {}^3C_1 \times {}^9C_1$$
$$= 8\times3\times9$$

Hence, required probability $= \dfrac{8\times3\times9}{{}^{20}C_3}$

$$= \frac{8\times3\times9\times6}{20\times19\times18}$$
$$= \frac{3\times6}{95} = \frac{18}{95}$$

(iii) Let E_3 be the event of getting one red and two white balls. Then,

$$n(E_3) = {}^8C_1 \times {}^3C_2 = 8\times3$$

Hence, required probability

$$= \frac{8\times3}{{}^{20}C_3} = \frac{8\times3}{\frac{20\times19\times18}{3\times2\times1}} = \frac{2}{95}$$

50. Total number of persons in the group $= 3+2+4 = 9$
Clearly, out of 9 persons 4 persons can be selected in ${}^9C_4$ ways.

Total number of possible ways $= {}^9C_4 = 126$

(i) Let E_1 be the event of getting 1 man, 1 woman and 2 children.

Then, $n(E_1) = {}^3C_1 \times {}^2C_1 \times {}^4C_2 = 3\times2\times6$

Hence, required probability $= \dfrac{3\times2\times6}{126} = \dfrac{2}{7}$

(ii) Let E_2 be the event of getting exactly 2 children.

Then, $n(E_2) = {}^4C_2 \times {}^5C_2 = 6\times10$

[$\because$ remaining two persons will be selected from 3 men and 2 women]

Hence, required probability $= \dfrac{6\times10}{126} = \dfrac{10}{21}$

(iii) Let E_3 be the event of getting 2 women.

Then $n(E_3) = C_2 \times {}^7C_2 = 1\times21 = 21$

Hence, required probability $= \dfrac{21}{126} = \dfrac{1}{6}$.

51. When a die is rolled, then we get 1 or 2 or 3 or 4 or 5 or 6.

$\therefore$ $S = \{1, 2, 3, 4, 5, 6\}$

It is given that

P(each odd number) $= 2 \times P$ (each even number)

Let P (each even number) $= x$. Then, we get

$P(1) = 2x$, $P(2) = x$, $P(3) = 2x$, $P(4) = x$, $P(5) = 2x$ and $P(6) = x$

We know that, $P(5) = 1$

$\therefore$ $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

$\Rightarrow$ $2x + x + 2x + x + 2x + x = 1$ $\Rightarrow$ $x = \dfrac{1}{9}$

Thus, $P(1) = P(3) = P(5) = \dfrac{2}{9}$

and $P(2) = P(4) = P(6) = \dfrac{1}{9}$

Now, $P(G) = P$ (getting 4 or 5 or 6)

$= P(4) + P(5) + P(6)$ **Ans.** $\dfrac{4}{9}$

52. Total number of ways of drawing two number $= {}^{50}C_2$

Prime numbers are 1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.

(i) Probability $= \dfrac{{}^{15}C_2}{{}^{50}C_2}$

(ii) Probability $= \dfrac{{}^{35}C_2}{{}^{50}C_2}$

(iii) Probability $= \dfrac{{}^{15}C_1 \times {}^{35}C_1}{{}^{50}C_2}$

Ans. (i) $\dfrac{21}{245}$ (ii) $\dfrac{17}{35}$ (iii) $\dfrac{3}{7}$

53. (i) (a) Thousand's place can be filled in 2 ways (5 or 7).

(b) The remaining 3 places can be filled by any of the digits 0, 1, 3, 5 and 7, as repetition of digits is allowed.

$\Rightarrow$ Number of 4-digits numbers $= (2\times5\times5\times5) - 1$ [$\because$ 5000 cannot be counted]

(c) If the number is divisible by 5, then its units place is filled by 0 or 5.

$\Rightarrow$ Number of 4-digits numbers divisible by $5 = (2\times5\times5\times2) - 1$

[here, repetition of digits is allowed]

Ans. $\dfrac{33}{83}$

(ii) (a) Thousand's place can be filled in 2 ways (5 or 7).

(b) The remaining 3 places can be filled in $4\times3\times2$ ways.

$\therefore$ Number of 4-digits numbers $= 2\times4\times3\times2$

(c) If the number is divisible by 5, then its units place is either 0 or 5.

(d) If units place is 0, then thousand, place can be filled in 2 ways and other two places can be filled in 3×2 ways.

(e) If units place is 5, then thousand's place can be filled in 1 way and other two places can be filled in 3×2 ways.

$\therefore$ Number of 4-digits numbers divisible by 5 $= 2\times3\times2 + 1\times3\times2$

Ans. $\dfrac{3}{8}$

|TOPIC 3|

Addition Rule of Probability

If A and B are two events associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

i.e. $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

It is known as *addition law of probability* for two events.

Also,

(i) when events A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$

(ii) when events A and B are mutually exclusive and exhaustive, then $P(A \cup B) = P(A) + P(B) = 1$.

e.g. If $P(A) = \frac{3}{5}$, $P(B) = \frac{2}{15}$

and $P(A \cap B) = \frac{1}{15}$, then by addition law of probability, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{3}{5} + \frac{2}{15} - \frac{1}{15} = \frac{9+2-1}{15} = \frac{10}{15} = \frac{2}{3}$$

$\Rightarrow \quad P(A \text{ or } B) = \frac{2}{3}$

Probability of the Event *A* or *B* or *C*

If A, B and C are three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Note

If A, B and C are mutually exclusive events,

i.e. $\quad A \cap B = \phi, B \cap C = \phi, A \cap C = \phi, A \cap B \cap C = \phi$.

Then, $\quad P(A \cup B \cup C) = P(A) + P(B) + P(C)$

Probability of Complementary Event

Let E be any event and $\overline{E}$ (or E') be its complementary event.

Then, $\quad P(\overline{E}) = 1 - P(E)$

Note

The odds in favour of occurrence of the event E are defined by $P(E) : P(\overline{E})$ and the odds against the occurrence of event E are defined by $P(\overline{E}) : P(E)$.

Important Results

1. For any two events A and B, $A \subseteq B \Rightarrow P(A) \le P(B)$
2. For any two events A and B,
   (i) $P(A - B) = P(A) - P(A \cap B)$
   or $P(A \cap \overline{B}) = P(A) - P(A \cap B)$
   (ii) $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$
3. $P(A \cap B) \le P(A)$ and $P(A \cap B) \le P(B)$

[if these conditions hold true, then events are said to be consistently defined]

Problem based on addition rule of probability

|TYPE 1|

PROBLEMS DIRECTLY BASED ON FORMULA

EXAMPLE |1| If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$.

Then, find P (not A and not B).

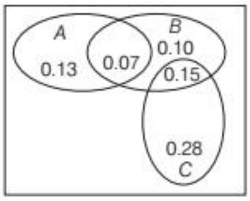
Sol. P (not A and not B) $= P(\overline{A} \cap \overline{B})$

$$= P(\overline{A \cup B}) = 1 - P(A \cup B)$$
$$= 1 - [P(A) + P(B) - P(A \cap B)]$$
$$= 1 - \left[\frac{1}{4} + \frac{1}{2} - \frac{1}{8}\right] = 1 - \left[\frac{2+4-1}{8}\right]$$
$$= 1 - \frac{5}{8} = \frac{3}{8}$$

EXAMPLE |2| The accompanying Venn diagram shows three events A, B and C and also the probabilities of the various intersections [for instance $P(A \cap B) = 0.07$]. Determine

(i) $P(A)$ (ii) $P(B \cap \overline{C})$
(iii) $P(A \cup B)$ (iv) $P(A \cap \overline{B})$ (v) $P(B \cap C)$
(vi) Probability of exactly one of the three event occurs.

[NCERT Exemplar]



Sol. (i) $P(A) = 0.13 + 0.07 = 0.20$

(ii) $P(B \cap \overline{C}) = P(B) - P(B \cap C)$

$= (0.07 + 0.10 + 0.15) - (0.15) = 0.17$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.20 + (0.07 + 0.10 + 0.15) - 0.07$

$= 0.20 + 0.25 = 0.45$

(iv) $P(A \cap \overline{B}) = P(A) - P(A \cap B) = 0.20 - 0.07 = 0.13$

(v) $P(B \cap C) = 0.15$

(vi) P (exactly one of the three events occurs)

$= P(A \text{ only}) + P(B \text{ only}) + P(C \text{ only})$

$= 0.13 + 0.10 + 0.28 = 0.51$

EXAMPLE |3| A and B are two events that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$. Find

(i) $P(A \cup B)$ (ii) $P(A' \cap B')$

(iii) $P(A \cap B')$ (iv) $P(B \cap A')$ **[NCERT]**

Sol. We have, $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.54 + 0.69 - 0.35 = 0.88$

(ii) $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B)$

$[\because P(E') = 1 - P(E)]$

$= 1 - 0.88 = 0.12$

(iii) $P(A \cap B') = P(A) - P(A \cap B) = 0.54 - 0.35 = 0.19$

(iv) $P(B \cap A') = P(B) - P(A \cap B) = 0.69 - 0.35 = 0.34$

EXAMPLE |4| The probability that at least one of the events A and B occurs is 0.6. If A and B occurs simultaneously with probability 0.2, then find $P(\overline{A}) + P(\overline{B})$. **[NCERT Exemplar]**

Sol. We have, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$

We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \quad 0.6 = P(A) + P(B) - 0.2$

$\Rightarrow \quad P(A) + P(B) = 0.6 + 0.2 = 0.8 \quad \text{...(i)}$

Now, $P(\overline{A}) + P(\overline{B}) = 1 - P(A) + 1 - P(B)$

$= 2 - (P(A) + P(B))$

$= 2 - 0.8$ [using Eq. (i)]

$= 1.2$

EXAMPLE |5| If A and B are any two events having $P(A \cup B) = \frac{1}{2}$ and $P(\overline{A}) = \frac{2}{3}$, then find $P(\overline{A} \cap B)$. **[NCERT Exemplar]**

Sol. We have, $P(A \cup B) = \frac{1}{2}$ and $P(\overline{A}) = \frac{2}{3}$

$\Rightarrow \quad 1 - P(A) = \frac{2}{3} \Rightarrow P(A) = 1 - \frac{2}{3} = \frac{1}{3}$

Also, we have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad \frac{1}{2} = \frac{1}{3} + P(B \cap \overline{A})$

$\Rightarrow \quad P(B \cap \overline{A}) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

$[\because P(\overline{A} \cap B) = P(B \cap \overline{A}) = P(B) - P(A \cap B)]$

EXAMPLE |6| Events E and F are such that P(not E or not F) $= 0.25$. State whether E and F are mutually exclusive. **[NCERT]**

Sol. We have, $P(\overline{E} \cup \overline{F}) = 0.25$

$\Rightarrow \quad P(\overline{E \cap F}) = 0.25 \quad [\because \overline{E \cap F} = \overline{E} \cup \overline{F}]$

$\Rightarrow \quad 1 - P(E \cap F) = 0.25$

$\Rightarrow \quad P(E \cap F) = 1 - 0.25 = 0.75 \neq 0$

$\therefore$ E and F are not mutually exclusive events.

EXAMPLE |7| If A, B and C are three mutually exclusive and exhaustive events of an experiment such that $3P(A) = 2P(B) = P(C)$, then find $P(A)$.

Sol. Let $3P(A) = 2P(B) = P(C) = p$. Then,

$$P(A) = \frac{p}{3},\ P(B) = \frac{p}{2} \text{ and } P(C) = p$$

Since, A, B and C are mutually exclusive and exhaustive.

$\therefore \quad P(A) + P(B) + P(C) = 1$

$\Rightarrow \quad \frac{p}{3} + \frac{p}{2} + p = 1 \Rightarrow \frac{2p + 3p + 6p}{6} = 1$

$\Rightarrow \quad 11p = 6 \Rightarrow p = \frac{6}{11}$

Hence, $\quad P(A) = \frac{p}{3} = \frac{6}{11} \times \frac{1}{3} = \frac{2}{11}$

EXAMPLE |8| Check whether the following probabilities $P(A)$ and $P(B)$ are consistently defined

(i) $P(A) = 0.5$, $P(B) = 0.7$, $P(A \cap B) = 0.6$

(ii) $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$ **[NCERT]**

Sol. (i) We have, $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B) = 0.6$.

We know that, $A \cap B \subseteq A$ and $A \cap B \subseteq B$

$\therefore$ $P(A \cap B) \leq P(A)$ and $P(A \cap B) \leq P(B)$

But for the given probabilities $P(A \cap B) \not\leq P(B)$.

So given probabilities are not consistently defined.

(ii) We have, $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.8$

Clearly, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.5 + 0.4 - 0.8 = 0.1$

Here, $P(A \cap B) \leq P(A)$

and $P(A \cap B) \leq P(B)$.

So, given probabilities are consistently defined.

EXAMPLE |9| Let A, B and C be three events such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.08$, $P(B \cap C) = 0.28$ and $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \geq 0.8$, then show that $P(A \cap C)$ lies in the interval $[0.23, 0.43]$.

Sol. We have, $\quad P(A \cup B \cup C) \geq 0.8$

$\therefore \quad 0.8 \leq P(A \cup B \cup C) \leq 1$

[$\because$ probability of occurrence of an event is always less than or equal to 1]

$\Rightarrow \quad 0.8 \leq P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C) \leq 1$

$$\Rightarrow \quad 0.8 \le 0.3 + 0.4 + 0.8 - 0.08 - 0.28 - P(A \cap C) + 0.09 \le 1$$

$$\Rightarrow \quad 0.8 \le 1.23 - P(A \cap C) \le 1$$

$$\Rightarrow \quad 0.8 - 1.23 \le -P(A \cap C) \le 1 - 1.23$$

[subtracting 1.23 from each terms]

$$\Rightarrow \quad -0.43 \le -P(A \cap C) \le -0.23$$

$$\Rightarrow \quad 0.23 \le P(A \cap C) \le 0.43$$

[multiplying each term by (-1)]

Hence, $P(A \cap C)$ lies in the interval [0.23, 0.43].

Hence proved.

| TYPE II |

PROBLEMS BASED ON COINS

EXAMPLE |10| Find the probability of getting atmost two tails or atleast two heads in a toss of three coins.

Sol. Clearly, the sample space

$$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$

Let A be the event of getting atmost two tails and B be the event of getting atleast two heads.

Then, $A = \{HHH, HHT, HTH, THH, TTH, THT, HTT\}$

and $B = \{HHT, HTH, THH, HHH\}$

$$P(A) = \frac{7}{8};\ P(B) = \frac{4}{8} \text{ and } P(A \cap B) = \frac{4}{8}$$

Now, required probability $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B) = \frac{7}{8} + \frac{4}{8} - \frac{4}{8} = \frac{7}{8}$$

| TYPE III |

PROBLEMS BASED ON DICE

EXAMPLE |11| A coin is tossed and a die is thrown. Find the probability that the outcome will be a head or a number greater than 4.

Sol. When a coin and a die are thrown, then sample space is

$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

$\therefore \quad n(S) = 12$

Let A : Event of getting a head.

$\Rightarrow \quad A = \{H1, H2, H3, H4, H5, H6\}$

$\therefore \quad n(A) = 6$

Then, $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{6}{12}$

B : Event of getting a number greater than 4.

$\Rightarrow \quad B = \{H5, H6, T5, T6\}$

$\therefore \quad n(B) = 4$

Then, $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{4}{12}$

Also, $A \cap B =$ event of getting a head and a number greater than $4 = \{H5, H6\}$

$\therefore \quad n(A \cap B) = 2$

Then, $\quad P(A \cap B) = \dfrac{2}{12}$

Now, required probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{12} + \frac{4}{12} - \frac{2}{12} = \frac{8}{12} = \frac{2}{3}$$

EXAMPLE |12| Two dice are tossed together. Find the probability of getting a doublet or a total of 6.

Sol. Let S be the sample space. Then, $n(S) = 36$.

Also, let A be the event of getting a doublet and B be the event of getting a total of 6.

Then, $\quad A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

and $\quad B = \{(1,5), (5,1), (2,4), (4,2), (3,3)\}$

$\Rightarrow \quad n(A) = 6,\ n(B) = 5$ and $n(A \cap B) = 1$

Then, $\quad P(A) = \dfrac{6}{36};\ P(B) = \dfrac{5}{36}$ and $P(A \cap B) = \dfrac{1}{36}$

Now, required probability $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B) = \frac{6}{36} + \frac{5}{36} - \frac{1}{36} = \frac{10}{36} = \frac{5}{18}$$

EXAMPLE |13| Find the probability of getting an even number on the first die or a total of 8 in a single throw of two dice.

Sol. Let S be the sample space. Then, $n(S) = 36$.

Also, let A be the event of getting an even number on the first die and B be the event of getting a total of 8.

Then, $$A = \begin{Bmatrix} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

$$B = \{(2,6), (6,2), (4,4), (3,5), (5,3)\}$$

and $\quad A \cap B = \{(2,6), (6,2), (4,4)\}$

$\Rightarrow \quad n(A) = 18,\ n(B) = 5$ and $n(A \cap B) = 3$

$\therefore \quad P(A) = \dfrac{18}{36},\ P(B) = \dfrac{5}{36}$ and $P(A \cap B) = \dfrac{3}{36}$

Now, required probability $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$$

EXAMPLE |14| Two dice are thrown together. What is the probability that sum of the numbers on the two faces is neither divisible by 3 nor by 4?

Sol. Let S be the sample space. Then, $n(S) = 36$.

Also, let A be the event that sum of numbers of two faces is divisible by 3 and B be the event that sum of numbers is divisible by 4, i.e. A be the event of getting the sum 3, 6, 9, 12 and B be the event of getting the sum 4, 8, 12.

Then,

$$A = \{(1,2), (2,1), (2,4), (4,2), (3,3), (1,5), (5,1), (4,5), (5,4), (6,3), (3,6), (6,6)\}$$

$$B = \{(2,2), (1,3), (3,1), (2,6), (6,2), (5,3), (3,5), (4,4), (6,6)\}$$

and $A \cap B = \{(6, 6)\}$

$\Rightarrow$ $n(A) = 12; n(B) = 9$ and $n(A \cap B) = 1$

$\therefore$ $P(A) = \frac{12}{36}, P(B) = \frac{9}{36}$ and $P(A \cap B) = \frac{1}{36}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{12}{36} + \frac{9}{36} - \frac{1}{36} = \frac{20}{36} = \frac{5}{9}$$

Now, required probability $= P(\overline{A} \cap \overline{B})$

$$= P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - \frac{5}{9} = \frac{4}{9}$$

|TYPE IV|

PROBLEM BASED ON PLAYING CARDS

EXAMPLE |15| A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that it is either a king or a spade.

Sol. Let S be the sample space associated with the given random experiment, then $n(S) = 52$.

Also, let A be the event that the drawn card is a king and B be the event that the drawn card is a spade.

Then, $n(A) = 4; n(B) = 13$ and $n(A \cap B) = 1$

[$\because A \cap B$ denotes the event that the drawn card is a king of spade.]

$\therefore$ $P(A) = \frac{4}{52}; P(B) = \frac{13}{52}$ and $P(A \cap B) = \frac{1}{52}$

Now, required probability $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

EXAMPLE |16| A card is drawn from a deck of 52 cards. Find the probability of getting a king or a heart or a red card. **[NCERT Exemplar]**

Sol. Let S be the sample space. Then, $n(S) = 52$

Let A, B and C be the events of getting a king, a heart and a red card, respectively.

Then, $n(A) = 4, n(B) = 13, n(C) = 26$

$\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

Clearly, $(A \cap B)$ is the event of getting a king among hearts.

$(B \cap C)$ is the event of getting a heart among red cards.

$(A \cap C)$ is the event of getting a king among red cards.

$(A \cap B \cap C)$ is the event of getting a king among heart and the red cards.

$\Rightarrow$ $n(A \cap B) = 1, n(B \cap C) = 13, n(A \cap C) = 2$

and $n(A \cap B \cap C) = 1$

$\therefore$ $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{52}$,

$$P(B \cap C) = \frac{n(B \cap C)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

$$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

and $P(A \cap B \cap C) = \frac{n(A \cap B \cap C)}{n(S)} = \frac{1}{52}$

Now, P (getting a king or heart or red card)

$= P(A \text{ or } B \text{ or } C) = P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$$= \frac{1}{13} + \frac{1}{4} + \frac{1}{2} - \frac{1}{52} - \frac{1}{26} - \frac{1}{4} + \frac{1}{52} = \frac{28}{52} = \frac{7}{13}$$

EXAMPLE |17| Four cards are drawn at a time from a pack of 52 playing cards. Find the probability of getting all the four cards of the same unit.

Sol. Let S be the sample space associated with the given random experiment. Then, $n(S) = {}^{52}C_4$.

Now, let A be the event of getting all spade cards,

B be the event getting all club cards,

C be the event getting all heart cards and

D be the event of getting all diamond cards.

Then, $n(A) = {}^{13}C_4; n(B) = {}^{13}C_4; n(C) = {}^{13}C_4$

and $n(D) = {}^{13}C_4$

$\Rightarrow$ $P(A) = \frac{{}^{13}C_4}{{}^{52}C_4}; P(B) = \frac{{}^{13}C_4}{{}^{52}C_4}; P(C) = \frac{{}^{13}C_4}{{}^{52}C_4}$

and $P(D) = \frac{{}^{13}C_4}{{}^{52}C_4}$.

Now, required probability $= P(A \cup B \cup C \cup D)$

[$\because$ A, B, C and D are mutually exclusive events]

$$= 4 \times \frac{{}^{13}C_4}{{}^{52}C_4} = \frac{44}{4165}$$

EXAMPLE |18| Find the probability that, when a hand of 7 cards is drawn from a well-shuffled deck of 52 cards, it contains

(i) all kings. (ii) 3 kings. (iii) at least 3 kings. **[NCERT]**

Sol. Let S be the sample space associated with the given random experiment. Then, $n(S) = {}^{52}C_7$

(i) Let A be the event that it contains all kings.

Then, $n(A) = {}^{4}C_4 \times {}^{48}C_3$

$$\Rightarrow \quad n(A) = \frac{{}^{48}C_3}{{}^{52}C_7} = \frac{\frac{48 \times 47 \times 46}{3 \times 2 \times 1}}{\frac{52 \times 51 \times 50 \times 49 \times 48 \times 47 \times 46}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}$$

$$= \frac{1}{7735} \qquad [\because {}^{4}C_4 = 1]$$

(ii) Let B be the event that it contains 3 kings.

Then, $n(B) = {}^4C_3 \times {}^{48}C_4$

$$\Rightarrow P(B) = \frac{4 \times {}^{48}C_4}{{}^{52}C_7} = \frac{\frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2 \times 1}}{\frac{52 \times 51 \times 50 \times 49 \times 48 \times 47 \times 46}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}$$

$$= \frac{9}{1547} \qquad [\because {}^4C_3 = {}^4C_1 = 4]$$

(iii) Let C be the event that it contains at least 3 kings.

Then, $P(C) = P(A \cup B)$

$= P(A) + P(B)$ [$\because A$ and B are mutually exclusive]

$$= \frac{1}{7735} + \frac{9}{1547} = \frac{46}{7735}$$

EXAMPLE |19| A card is drawn from an ordinary pack and a gambler bets that it is spade or an ace.

What are the odds against the winning his bet?

Sol. Let S be the sample space associated with the given random experiment. Then, $n(S)=52$.

Now, let A be the event of getting a spade and B be the event of getting an ace. $A \cap B$ denote the event of getting an ace of spade.

Then, $n(A) = 13$ and $n(B) = 4$

$n(A \cap B) = 1$

Thus, we have $P(A) = \frac{13}{52}$; $P(B) = \frac{4}{52}$ and $P(A \cap B) = \frac{1}{52}$

Now, P (winning the bet) $= P$ (getting a spade or an ace)

$= P(A \cap B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

$$\Rightarrow P \text{ (not winning the bet)} = 1 - \frac{4}{13} = \frac{9}{13}$$

Hence, odds against the winning his bet

$$= \frac{P(\text{not winning the bet})}{P(\text{winning the bet})} = \frac{\frac{9}{13}}{\frac{4}{13}} = \frac{9}{4} = 9:4$$

|TYPE V|

PROBLEMS BASED ON SELECTING NUMBERS

EXAMPLE |20| An integer is chosen at random from first two hundred natural numbers. What is the probability that the integer chosen is divisible by 6 or 8?

Sol. Let S be the sample space associated with the given random experiment.

Then, $n(S)=200$

Now, let A be the event that the chosen number is divisible by 6 and B be the event that the chosen number is divisible by 8.

Then, $A = \{6, 12, 18, \ldots, 198\}$

and $B = \{8, 16, 24, \ldots, 192\}$

$\Rightarrow n(A) = 33$ and $n(B)=24$

[to find $n(A)$ and $n(B)$, divide 198 by 6 and 192 by 8]

Also, $A \cap B = \{24, 48, 72, \ldots, 192\}$

[$\because A \cap B$ denotes the event that the chosen number is divisible by LCM (6,8)]

Now, $P(A) = \frac{33}{200}$; $P(B) = \frac{24}{200}$ and $P(A \cap B) = \frac{8}{200}$

$\therefore$ P (chosen integer is divisible by 6 or 8)

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{33}{200} + \frac{24}{200} - \frac{8}{200} = \frac{49}{200}$$

EXAMPLE |21| Tickets numbered 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is a multiple of 3 or 7?

Sol. Let S be the sample space associated with the given random experiment. Then, $n(S)=20$.

Now, let A be the event of getting a ticket bearing number which is a multiple of 3 and B be the event of getting a ticket bearing a number which is a multiple of 7.

Then, $A = \{3, 6, 9, 12, 15, 18\}$ and $B = \{7, 14\}$

$\Rightarrow n(A) = 6$, $n(B)=2$ and $n(A \cap B) = 0$

$\therefore P(A) = \frac{6}{20}$ and $P(B) = \frac{2}{20}$

Now, required probability $= P(A \cup B) = P(A) + P(B)$

[$\because A$ and B are mutually exclusive events]

$$= \frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5}$$

EXAMPLE |22| Three numbers are chosen from 1 to 20. Find the probability that they are not consecutive.

[NCERT Exemplar]

Sol. Let S be the sample space associated with the given experiment. Then,

$$n(S) = {}^{20}C_3 = 1140$$

Now, let E be the event that the chosen numbers are consecutive.

Then, $\overline{E}$ be the event that the chosen numbers are not consecutive.

Thus, $\overline{E} = \{(1, 2, 3), (2, 3, 4), (3, 4, 5), \ldots, (18, 19, 20)\}$

$\Rightarrow n(\overline{E}) = 18 \Rightarrow P(\overline{E}) = \frac{18}{1140} = \frac{3}{190}$

Now, $P(E) = 1 - P(\overline{E}) = 1 - \frac{3}{190} = \frac{187}{190}$

Hence, the required probability is $\frac{187}{190}$.

EXAMPLE |23| A number is chosen at random from the numbers ranging from 1 to 50. What is the probability that the number chosen is a multiple of 2 or 3 or 10?

Sol. Let S be the sample space associated with the given random experiment.

Then, $n(S)=50$.

Now, let A be the event that the chosen number is a multiple of 2, B be the event that the chosen number is a multiple of 3 and C be the event that the chosen number is a multiple of 10.

Then, $A = \{2, 4, 6, \ldots, 48, 25\}$

$B = \{3, 6, 9 \ldots, 48\}$ and $C = \{10, 20, \ldots, 50\}$

$A \cap B$ denotes the event that chosen number is multiple of $6 = \text{LCM}(2, 3)$

$B \cap C$ denotes the event that chosen number is a multiple of $30 = \text{LCM}(3, 10)$

$A \cap C$ denotes the event that chosen number is a multiple of $10 = \text{LCM}(2, 10)$

and $A \cap B \cap C$ denotes the event that chosen number is a multiple of $30 = \text{LCM}(2, 3, 10)$

$\therefore \quad A \cap B = \{6, 12, \ldots, 48\}, n(B) = 16,$

$B \cap C = \{30\}, A \cap C = \{10, 20 \ldots, 50\}$

and $\quad A \cap B \cap C = \{30\}$

$\Rightarrow n(A) = 25; n(B) = \{16\}; n(C) = 5; n(A \cap B) = 8,$

$n(B \cap C) = 1, n(A \cap C) = 5$ and $n(A \cap B \cap C) = 1$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{25}{50}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{16}{50}$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{5}{50}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{8}{50},$$

$$P(B \cap C) = \frac{n(B \cap C)}{n(S)} = \frac{1}{50},$$

$$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{5}{50}$$

$$\text{and} \quad P(A \cap B \cap C) = \frac{n(A \cap B \cap C)}{n(S)} = \frac{1}{50}$$

Now, required probability

$= P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B)$

$- P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$$= \frac{25}{50} + \frac{16}{50} + \frac{5}{50} - \frac{8}{50} - \frac{1}{50} - \frac{5}{50} + \frac{1}{50} = \frac{25+8}{50} = \frac{33}{50}$$

Hence, required probability is $\frac{33}{50}$.

|TYPE VI|

PROBLEMS BASED ON SELECTING THINGS FROM A BAG/BOX CONTAINING DIFFERENT TYPES OF THINGS

EXAMPLE |24| *A basket contains 20 apples and 10 oranges out of which 5 apples and 3 oranges are defective. If a person takes out 2 at random, then what is the probability that either both are apples or both are good?*

Sol. Let S be the sample space associated with the given experiment. Then, $n(S) = {}^{30}C_2$.

[$\because$ total number of fruits $= 20 + 10 = 30$]

Now, let A be the event that both are apples and B be the event that both are good fruits. $A \cap B$ denotes the event that both are good apples.

Then, $n(A) = {}^{20}C_2$ and $n(B) = {}^{22}C_2$.

[$\because$ number of good fruits $= 15 + 7 = 22$]

$n(A \cap B) = {}^{15}C_2$

[$\because$ number of good apples $= 15$]

$$\therefore \quad P(A) = \frac{{}^{20}C_2}{{}^{30}C_2}, P(B) = \frac{{}^{22}C_2}{{}^{30}C_2} \text{ and } P(A \cap B) = \frac{{}^{15}C_2}{{}^{30}C_2}$$

Now, required probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{{}^{20}C_2}{{}^{30}C_2} + \frac{{}^{22}C_2}{{}^{30}C_2} - \frac{{}^{15}C_2}{{}^{30}C_2}$$

$$= \frac{190 + 231 - 105}{{}^{30}C_2} = \frac{316}{435}$$

EXAMPLE |25| A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn, there is atleast one ball of each colour.

Sol. Given, total number of red balls $= 6$

Total number of white balls $= 4$

and total number of black balls $= 5$

$\therefore$ Total number of balls $= 15$

Let S be the sample space associated with the random experiment. Then, $n(S) = {}^{15}C_4 = 1365$

Here, atleast one ball of each colour can be drawn in one of the following ways :

(i) 1 red, 1 white and 2 black balls

(ii) 2 red, 1 white and 1 black balls

(iii) 1 red, 2 white and 1 black balls

Now, let A be the event of getting 1 red, 1 white and 2 black balls; B be the event of getting 2 red, 1 white and 1 black balls and C be the event of getting 1 red, 2 white and 1 black balls.

Then, $\quad n(A) = {}^6C_1 \times {}^4C_1 \times {}^5C_2 = 6 \times 4 \times 10 = 240$

$n(B) = {}^6C_2 \times {}^4C_1 \times {}^5C_1 = 15 \times 4 \times 5 = 300$

and $\quad n(C) = {}^6C_1 \times {}^4C_2 \times {}^5C_1 = 6 \times 6 \times 5 = 180$

$$\therefore \quad P(A) = \frac{240}{1365}; P(B) = \frac{300}{1365} \text{ and } P(C) = \frac{180}{1365}$$

Now, required probability

$= P(A \cup B \cup C) = P(A) + P(B) + P(C)$

[$\because$ A, B and C are mutually exclusive events]

$$= \frac{240}{1365} + \frac{300}{1365} + \frac{180}{1365} = \frac{720}{1365} = \frac{48}{91}$$

|TYPE VII|

MISCELLENEOUS PROBLEMS

EXAMPLE |26| Probability that Ram passed in Mathematics is $\frac{2}{3}$ and the probability that he passed in English is $\frac{4}{9}$. If the probability of passing in both subjects is $\frac{1}{4}$, then what is the probability that Ram will pass in atleast one of these subjects?

Sol. Let M be the event that Ram passed in Mathematics, E be the event that Ram passed in English and $M \cap E$ be the event that Ram passed in both subjects

Now, P (Ram pass in atleast one subject) $= P(M \cup E)$

$$= P(M) + P(E) - P(M \cap E) = \frac{2}{3} + \frac{4}{9} - \frac{1}{4}$$

$$= \frac{24 + 16 - 9}{36} = \frac{31}{36}$$

EXAMPLE |27| In class XI of a school, 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, then find the probability that he will be studying Mathematics or Biology. **[NCERT]**

Sol. Let M be the event that selected student studies Mathematics and B be the event that selected student studies Biology. Then, $P(M) = 40\% = 0.4$

$P(B) = 30\% = 0.3$ and $P(M \cap B) = 10\% = 0.1$

Now, P (he will study Mathematics or Biology)

$= P(M \cup B) = P(M) + P(B) - P(M \cap B) = 0.4 + 0.3 - 0.1 = 0.6$

Hence, the required probability is 0.6.

EXAMPLE |28| Probability that a truck stopped at a roadblock will have faulty brakes or badly worn tires are 0.23 and 0.24, respectively. Also, the probability is 0.38 that a truck stopped at the roadblock will have faulty brakes and/or badly working tires. What is the probability that a truck stopped at this roadblock will have faulty breaks as well as badly worn tires?

Sol. Let B be the event that a truck stopped at the roadblock will have faulty breaks and T be the event that it will have badly worn tires. Then, we have

$$P(B) = 0.23,\ P(T) = 0.24 \text{ and } P(B \cup T) = 0.38$$

Now, consider, $P(B \cup T) = P(B) + P(T) - P(B \cap T)$

$\Rightarrow \quad 0.38 = 0.23 + 0.24 - P(B \cap T)$

$\Rightarrow \quad P(B \cap T) = 0.23 + 0.24 - 0.38 = 0.09$

Hence, the probability that a truck stopped at the roadblock will have faulty breaks as well as badly worn tires is 0.09.

EXAMPLE |29| The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, then what is the probability of passing the Hindi examination? **[NCERT]**

Sol. Let events H and E denote the students that, passing Hindi and English examination, respectively.

Given, probability of passing both examinations

$$= P(H \cap E) = 0.5$$

and probability of passing neither examinations

$$= P(H' \cap E') = P(H \cup E)' = 0.1$$

Then, $P(H \cup E) = 1 - P(H \cup E)' = 1 - 0.1 = 0.9$

Also, it is given, that probability of passing English examination,

$$P(E) = 0.75$$

Now, consider $P(H \cup E) = P(H) + P(E) - P(H \cap E)$

$\Rightarrow \quad 0.9 = P(H) + 0.75 - 0.5$

$\Rightarrow \quad 0.9 = P(H) + 0.25$

$\Rightarrow \quad P(H) = 0.9 - 0.25 = 0.65$

Hence, probability of passing Hindi examination is 0.65.

EXAMPLE |30| In an interview for a job in call center, 5 boys and 3 girls appeared. If 4 persons are to be selected at random from this group, then find the probability that 3 boys and 1 girl or 1 boy and 3 girls are selected.

Sol. Given, number of boys = 5 and number of girls = 3

Let A : Selection of 3 boys and 1 girl

B : Selection of 1 boy and 3 girls

$$\therefore \quad P(A) = \frac{{}^5C_3 \times {}^3C_1}{{}^8C_4} = \frac{\frac{5 \times 4}{2 \times 1} \times 3}{\frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}} = \frac{30}{70} = \frac{3}{7}$$

and $$P(B) = \frac{{}^5C_1 \times {}^3C_3}{{}^8C_4} = \frac{5 \times 1}{\frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}} = \frac{5}{70} = \frac{1}{14}$$

Hence, the required probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - 0 = \frac{3}{7} + \frac{1}{14} = \frac{7}{14} = \frac{1}{2}$$

EXAMPLE |31| The probability that a student will receive A, B, C or D grade are 0.40, 0.35, 0.15 and 0.10, respectively. Find the probability that a student will receive

(i) B or C grade (ii) atleast C grade.

Sol. Let E_1, E_2, E_3 and E_4 denote respectively the events that a student will receive A, B, C and D grades. Then, we have

$$P(E_1) = 0.40,\ P(E_2) = 0.35,\ P(E_3) = 0.15 \text{ and } P(E_4) = 0.10$$

(i) P (getting B or C grade) $= P(E_2 \cup E_3) = P(E_2) + P(E_3)$

$$= 0.35 + 0.15 = 0.5$$

[$\because E_2$ and E_3 are mutually exclusive events]

(ii) P (getting atleast C grade) = P (getting C or D grade)

$= P(E_3 \cup E_4) = P(E_3) + P(E_4)$

$= 0.15 + 0.10 = 0.25$

EXAMPLE |32| Four candidates A, B, C, D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B, and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, what are the probabilities that

(i) C will be selected? (ii) A will not be selected?

Sol. Let E_1, E_2, E_3 and E_4 denote respectively the events that the person A, B, C and D is selected.

Also, let the probability of D selected be p, i.e. $P(E_4) = p$.

Then, according to given condition we have

$P(E_1) = 2P(E_2)$ and $P(E_2) = P(E_3) = 2P(E_4)$

$\Rightarrow \quad P(E_1) = 2 \times 2p,\ P(E_2) = 2p$ and $P(E_3) = 2p$

$\Rightarrow \quad P(E_1) = 4p,\ P(E_2) = 2p$ and $P(E_3) = 2p$

Since, E_1, E_2, E_3 and E_4 are mutually exclusive and exhaustive events, therefore we have

$P(E_1) + P(E_2) + P(E_3) + P(E_4) = 1$

$\Rightarrow \quad 4p + 2p + 2p + p = 1$

$\Rightarrow \quad 9p = 1 \Rightarrow p = \frac{1}{9}$

(i) $P(C \text{ selected}) = P(E_3) = 2p = \frac{2}{9}$

(ii) Now, $P(A \text{ selected}) = P(E_1) = 4p = \frac{4}{9}$

$\therefore$ $P(A$ is not selected$) = 1 - P(A$ selected$)$

$= 1 - \frac{4}{9} = \frac{5}{9}$

EXAMPLE |33| A box contains 100 bolts and 50 nuts. It is given that 50% bolts and 50% nuts are rusted. Two objects are selected from the box at random. Find the probability that either both are bolts or both are rusted.

Sol. Given, total number of bolts = 100

and total number of nuts = 50

Now, total number of rusted objects

$= 50\%$ of $100 + 50\%$ of 50

$= \frac{1}{2} \times 100 + \frac{1}{2} \times 50 = 50 + 25 = 75$

Clearly, total number of objects $= 100 + 50 = 150$

Let S be the sample space associated with the given experiment. Then, $n(S) = {}^{150}C_2$

Now, let A be the event that both objects are bolts and B be the event that both objects are rusted.

Then, $n(A) = {}^{100}C_2$ and $n(B) = {}^{75}C_2$

$A \cap B$ denotes the event that both objects are rusted bolts.

$\therefore \quad n(A \cap B) = {}^{50}C_2$

[$\because$ number of rusted bolts = 50% of 100 = 50]

Now, $P(A) = \frac{n(A)}{n(S)} = \frac{{}^{100}C_2}{{}^{150}C_2}$; $P(B) = \frac{n(B)}{n(S)} = \frac{{}^{75}C_2}{{}^{150}C_2}$

and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{{}^{50}C_2}{{}^{150}C_2}$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B) = \frac{{}^{100}C_2 + {}^{75}C_2 - {}^{50}C_2}{{}^{150}C_2}$

$= \frac{4950 + 2775 - 1225}{11175} = \frac{6500}{11175} = 0.58$

Hence, the required probability is 0.58.

EXAMPLE |34| Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, then what is the probability that

(i) you both enter the same section?

(ii) you both enter the different sections?

Sol. Let there be two sections A and B of 40 and 60 students, respectively.

(i) Let both students enter the section A.

So, 38 students are to selected out of 98, since two particular students are already in section A.

$\therefore$ Number of ways of selecting 38 students out of $98 = {}^{98}C_{38}$ ways

Number of exhaustive cases of selecting 40 students out of $100 = {}^{100}C_{40}$ ways

$\therefore$ P (both students enter section A)

$= \frac{{}^{98}C_{38}}{{}^{100}C_{40}} = \frac{98!}{38!\,60!} \div \frac{100!}{40!\,60!}$

$= \frac{98!}{38!\,60!} \times \frac{40!\,60!}{100!} = \frac{40 \times 39}{100 \times 99} = \frac{26}{165}$

If both students enter the section B. Then, the number of ways of selecting 58 students out of $98 = {}^{98}C_{58}$ ways.

Total number of ways of selecting 60 students out of $100 = {}^{100}C_{60}$ ways.

$\therefore$ Probability that two students enter section B

$= \frac{{}^{98}C_{58}}{{}^{100}C_{60}} = \frac{98!}{58!\,40!} \div \frac{100!}{60!\,40!} \quad \left[\because {}^nC_r = \frac{n!}{r!(n-r)!}\right]$

$= \frac{98!}{58!\,40!} \times \frac{60!\,40!}{100!} = \frac{60 \times 59}{100 \times 99} = \frac{59}{165}$

$\therefore$ P (that two particular students enter either section A or B)

$= \frac{26}{165} + \frac{59}{165} = \frac{85}{165} = \frac{17}{33}$

(ii) The probability that they enter different sections

$= 1 - P$ (that two particular students enter either section A or B)

$= 1 - \frac{17}{33} = \frac{16}{33}$

TOPIC PRACTICE 3

OBJECTIVE TYPE QUESTIONS

1 If A and B are mutually exclusive events, then
(a) $P(A) \le P(\overline{B})$ (b) $P(A) \ge P(\overline{B})$
(c) $P(A) < P(\overline{B})$ (d) None of these

2 Probability that a truck stopped at a roadblock will have faulty brakes and badly worn tires, are 0.23 and 0.24, respectively. Also, the probability is 0.38 that a truck stopped at the roadblock will have faulty brakes or badly worn tires. Then, the probability that a truck stopped at this roadblock will have faulty breaks as well as badly worn tires, is ...K... . Here, K refers to
(a) 0.04 (b) 0.07 (c) 0.06 (d) 0.09

3 If A, B and C are three mutually exclusive and exhaustive events of an experiment such that $4P(A) = 2P(B) = P(C)$, then $P(A)$ is equal to ...K... . Here, K refers to
(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{5}{7}$ (d) $\frac{6}{7}$

4 In an essay competition, the odds in favour of competitors P, Q, R and S are 1 : 2, 1: 3, 1 : 4 and 1 : 5 respectively. Then, the probability that one of them wins the competition, is
(a) $\frac{112}{120}$ (b) $\frac{113}{120}$ (c) $\frac{114}{120}$ (d) $\frac{115}{120}$

5 If $P(B) = \frac{3}{4}$, $P(A \cap B \cap \overline{C}) = \frac{1}{3}$ and $P(\overline{A} \cap B \cap \overline{C}) = \frac{1}{3}$, then $P(B \cap C)$ equals
(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{15}$ (d) $\frac{1}{9}$

VERY SHORT ANSWER Type Questions

6 If A and B are two events such that $P(A)=\frac{1}{4}$, $P(B)=\frac{1}{2}$ and P (A and B) $= \frac{1}{8}$. Then, find P (A or B).

7 If $\frac{2}{11}$ is the probability of an event A, then what is the probability of the event 'not A'?

8 Given, $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find $P(A$ or $B)$, if A and B are mutually exclusive events.

SHORT ANSWER Type I Questions

9 A and B are two events, such that $P(A) = 0.42$, $P(B) = 0.48$ and $P(A$ and $B) = 0.16$. Determine
(i) P(not A) (ii) P(not B)
(iii) $P(A$ or $B)$ **[NCERT]**

10 If E and F are events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E$ and $F) = \frac{1}{8}$, then find
(i) $P(E$ or $F)$ (ii) P (not E and not F)

11 If A and B are mutually exclusive events, such that $P(A) = 0.28$ and $P(B) = 0.38$, then find
(i) $P(A \cup B)$ (ii) $P(A \cap B)$
(iii) $P(A \cap B')$ (iv) $P(A' \cap B')$

12 If E_1, E_2 and E_3 are three mutually exclusive and exhaustive events of an experiment such that $2P(E_1) = 3\,P(E_2) = P(E_3)$, then find $P(E_1)$.

13 Check whether the following probabilities $P(A) = 0.3$, $P(B) = 0.65$ and $P(A \cup B) = 7$ are consistently defined.

14 The probabilities of happening of two events A and B are 0.25 and 0.50, respectively. If the probablity of happening of A and B together is 0.14, then find the probability that neither A nor B occurs.

15 The probability of an event A occurring is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B occurs.

16 A and B are mutually exclusive events of an experiment. If P (not A) = 0.65, $P(A \cup B)=0.65$ and $P(B)=p$, then find the value of p.

17 In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing atleast one of them is 0.95. What is the probability of passing both? **[NCERT]**

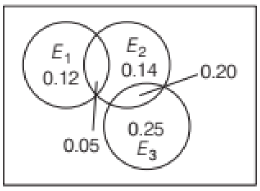
18 The probability that atleast one of the events A and B occurs is 0.7. If A and B occurs simultaneously with probability 0.35, then find $P(\overline{A}) + P(\overline{B})$.

19 Events E_1 and E_2 are such that P(not E_1 or not E_2) = 0.35. State whether E_1 and E_2 are mutually exclusive.

20 In any school examination, probability that Mohan passed in Chemistry is $\frac{1}{3}$ and the probability that he passed in Physics is $\frac{2}{5}$. If the probability of passing in both subjects is $\frac{1}{6}$, then what is the probability that Mohan will pass in atleast one of these subjects?

SHORT ANSWER Type II Questions

21 The probabilities of three events shown in the Venn diagram is shown below



Determine

(i) $P(E_2)$ (ii) $P(E_2 \cap \overline{E}_3)$
(iii) $P(E_1 \cup E_2)$ (iv) $P(E_1 \cap \overline{E}_2)$

22 Let A, B and C are three events, such that
$P(A) = 0.4$, $P(B) = 0.42$, $P(C) = 0.7$,
$P(A \cap B) = 0.2$, $P(A \cap C) = 0.32$ and
$P(A \cap B \cap C) = 0.07$. If $P(A \cup B \cup C) \geq 0.7$, then show that $P(B \cap C)$ lies in the interval $[0.07, 0.37]$.

23 Find the probability of getting atmost two heads or atleast two tails in a toss of three coins.

24 The probability that a person will get an electric contract is $\frac{2}{5}$ and the probability that he will not get plumbing contract is $\frac{4}{7}$. If the probability of getting atleast one contract is $\frac{2}{3}$, then what is the probability that he will get both?

25 In a large metropolitan area, the probabilities are 0.87, 0.36, 0.30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set, or both kinds of sets. What is the probability that a family owns either anyone or both kinds of sets? **[NCERT Exemplar]**

26 Two unbiased dice are thrown. Find the probability that neither a doublet nor a total of 10 will appear.

27 Two dice are thrown together. What is the probability that sum of the numbers on the two faces is divisible by 3 or 4?

28 If $\frac{5}{14}$ is the probability of occurrence of an event, find

(i) the odds in favour of its occurrence.
(ii) the odds against its occurrence.

29 In a drawing competition, the odds in favour of competitors A, B, C and D are 1:2, 1:3, 1:4 and 1:5, respectively. Find the probability that one of them wins the competition.

30 From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability that it is either a heart or a queen.

31 One number is chosen from numbers 1 to 200. Find the probability that it is divisible by 4 or 6.

32 A coin is tossed and a die is thrown. Find the probability that the outcomes will be a tail or a number greater than 3.

33 Find the probability of getting an odd number of the first die or a total of 7 in a single throw of two dice.

34 Two dice are thrown together. What is the probability that sum of the numbers on the two faces is neither divisible by 3 nor by 5?

35 Two dice are thrown. Find

(i) odds in favour of getting the sum 5.
(ii) the odd against getting the sum 6.

36 If a person visits his dentist, suppose the probability that he will have his teeth cleaned is 0.48, the probability that he will have a cavity filled is 0.25, the probability that he will have a tooth extracted is 0.20, the probability that he will have a teeth cleaned and a cavity filled is 0.09, the probability that he will have his teeth cleaned and a tooth extracted is 0.12, the probability that he will have a cavity filled and a tooth extracted is 0.07, and the probability that he will have his teeth cleaned, a cavity filled, and a tooth extracted is 0.03. What is the probability that a person visiting his dentist will have atleast one of these things done to him? **[NCERT Exemplar]**

37 Find the probability that, when a hand of 5 cards is drawn from a well-shuffled deck of 52 cards, it contains

(i) all queens. (ii) 3 queens.

(iii) atleast 3 queens.

38 Tickets numbered 1 to 30 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is a multiple of 2 or 5?

39 Let A, B and C be three events. If the probability of occuring exactly one event out of A and B is $1-x$, out of B and C is $1-2x$, out of C and A is $1-x$, and that of occuring three events simultaneously is x^2, then prove that the probability that at least one out of A, B, C will occur is greater than 1/2.

40 Suppose an integer from 1 through 1000 is chosen at random. Find the probability that the integer is a multiple of 2 or a multiple of 9. **[NCERT Exemplar]**

41 In a race, the odds in favour of horses P, Q, R and S are $1:2$, $1:3$, $1:4$ and $1:5$, respectively. Find the probability that one of them wins the race.

42 If the odds against winning a race of three horses are respectively $3:1$, $4:1$ and $5:1$, then what is the probability that one of these horses will win?

43 A basket contains 15 guava and 12 banana, out of which 5 guava and 7 banana are defective. If a person takes out 3 at random, then what is the probability that either all are guava or all are good?

44 A card is drawn from an ordinary pack and a gambler bets that it is a diamond or a jack. What are the odds favour in the winning his bet?

45 In higher secondary school, 30% students study Economics and 40% study Accounts. 20% of the class study both Economics and Accounts. If a student is selected at random from the class, then find the probability that he will be studying Economics or Accounts.

46 The probability that a student will receive A, B, C and D grades are 0.50, 0.40, 0.20 and 0.15, respectively. Find the probability that a student will receive

(i) B or C grade. (ii) atleast C grade.

47 A box contains 4 red, 5 white and 6 black balls. A person draws 4 balls from the box at random. Find the probability of selecting atleast one ball of each colour.

48 A die has two faces each with number '1', three faces each with number '2' and one faces with number '3'. If die is rolled once, determine

(i) $P(2)$ (ii) $P(1 \text{ or } 3)$ (iii) $P(\text{not } 3)$.

49 A bag contains 5 white and 7 black balls and a man draw 4 balls at random. What are the odds against these being all black?

LONG ANSWER Type Questions

50 A number is chosen at random from the numbers ranging from 20 to 50. What is the probability that the number chosen is a multiple of 3 or 5 or 7?

51 A card is drawn from a well-shuffled deck of 52 cards. Find

(i) the odds in favour of getting a face card.

(ii) the odds against getting a spade.

52 A box contains 200 bolts and 70 nuts. It is given that 60% bolts and 40% nuts are rusted. Two objects are selected at random from the box. Find the probability that either both are bolts or both are rusted.

53 If A and B are mutually exclusive events, such that $P(A) = 0.35$ and $P(B) = 0.45$, then find

(i) $P(A')$ (ii) $P(B')$

(iii) $P(A \cup B)$ (iv) $P(A \cap B)$

(v) $P(A \cap B')$ (vi) $P(A' \cap B')$

[NCERT Exemplar]

54 In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, then find the probability that

(i) the student opted for NCC or NSS.

(ii) the student has opted for neither NCC nor NSS.

(iii) the student has opted NSS but not NCC.

HINTS & ANSWERS

1. (a) Given that, A and B are two mutually exclusive events Then,

$$P(A \cup B) = P(A) + P(B) \qquad [\because (A \cap B) = \phi]$$

$$P(A) + P(B) \le 1$$

$$P(A) + 1 - P(\overline{B}) \le 1$$

$$P(A) \le P(\overline{B})$$

2. (d) Let B be the event that a truck stopped at the roadblock will have faulty brakes and T be the event that it will have badly worn tires.

We have $P(B) = 0.23$, $P(T) = 0.24$ and $P(B \cup T) = 0.38$
and $P(B \cup T) = P(B) + P(T) - P(B \cap T)$
So $0.38 = 0.23 + 0.24 - P(B \cap T)$
$\Rightarrow P(B \cap T) = 0.09$

3. (a) Let $4P(A) = 2P(B) = P(C) = p$ which gives $P(A) = \frac{p}{4}$,
$P(B) = \frac{p}{2}$ and $P(C) = p$

Now, since A, B, C are mutually exclusive and exhaustive events, we have

$$P(A) + P(B) + P(C) = 1$$

$$\Rightarrow \quad p = \frac{4}{7}$$

Hence, $P(A) = \frac{p}{4} = \frac{1}{7}$

4. (c) Let A, B, C, D be the events that the competitors P, Q, R and S respectively win the competition.

Then, $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{5}$ and $P(D) = \frac{1}{6}$

Since, only one competitors can win the competition.
Therefore, A, B, C, D are mutually exclusive events.

$\therefore$ Required probability $= P(A \cup B \cup C \cup D)$
$= P(A) + P(B) + P(C) + P(D)$
$= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{114}{120}$

5. (a) We have, $P(\overline{A} \cap B \cap \overline{C}) = \frac{1}{3}$

$$\Rightarrow \quad P((B \cap \overline{C}) \cap \overline{A}) = \frac{1}{3}$$

$$\Rightarrow \quad P(B \cap \overline{C}) - P((B \cap \overline{C}) \cap A) = \frac{1}{3}$$

$$[\because P(X \cap \overline{Y}) = P(X) - P(X \cap Y)]$$

$$\Rightarrow \quad P(B \cap \overline{C}) - P(A \cap B \cap \overline{C}) = \frac{1}{3}$$

$$\Rightarrow \quad P(B \cap \overline{C}) - \frac{1}{3} = \frac{1}{3}$$

$$\Rightarrow \quad P(B \cap \overline{C}) = \frac{2}{3}$$

$$\Rightarrow \quad P(B) - P(B \cap C) = \frac{2}{3}$$

Now, $P(B \cap C) = P(B) - \frac{2}{3} = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

6. $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$ **Ans.** $\frac{5}{8}$

7. $P(\text{not } A) = P(\overline{A}) = 1 - P(A)$ **Ans.** $\frac{9}{11}$

8. $P(A \cup B) = P(A) + P(B)$ **Ans.** $\frac{4}{5}$

9. (i) $P(\text{not } A) = 1 - P(A)$ **Ans.** 0.58
(ii) $P(\text{not } B) = 1 - P(B)$ **Ans.** 0.52
(iii) $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$ **Ans.** 0.74

10. (i) $P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$
(ii) $P(\overline{E} \cap \overline{F}) = P(\overline{E \cap F}) = 1 - P(E \cap F)$

Ans. (i) $\frac{5}{8}$ (ii) $\frac{3}{8}$

11. (i) $P(A \cup B) = P(A) + P(B)$
(ii) $P(A \cap B) = 0$
(iii) $P(A \cap B') = P(A) - P(A \cap B)$
(iv) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

Ans. (i) 0.66 (ii) 0
(iii) 0.28 (iv) 0.34

12. Solve as Example 7 **Ans.** $\frac{3}{11}$

13. Solve as Example 8 part (ii). **Ans.** Yes.

14. $P(A \cup B) = 0.25 + 0.50 - 0.14 = 0.61$
$\therefore P(\text{neither } A \text{ nor } B) = P(\overline{A} \cap \overline{B})] = 1 - P(A \cup B)$
Ans. 0.39

15. $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$
$= 1 - P(A \cup B) = 1 - [P(A) + P(B)]$ **Ans.** 0.2

16. Use $P(A \cup B) = P(A) + P(B)$, for mutually exclusive events.
Ans. 0.3

17. Let E and F denote the events that, students passing the first and second examination, respectively.
Then, $P(E) = 0.8$, $P(F) = 0.7$ and $P(E \cup F) = 0.95$
Ans. 0.55

18. Solve as Example 4. **Ans.** 1.05

19. Solve as Example 6. **Ans.** 0.65

20. Solve as Example 26. **Ans.** $\frac{17}{30}$

21. Solve as Example 2.
Ans. (i) 0.39 (ii) 0.19
(iii) 0.31 (iv) 0.12

22. Solve as Example 9.

23. Solve as Example 10. **Ans.** $\frac{7}{8}$

24. Let A be the event that the person get an electric contract. and B be the event that the person get an plumbing contract.
Then, $P(A) = \frac{2}{5}$, $P(\overline{B}) = \frac{4}{7}$ and $P(A \cup B) = \frac{2}{3}$ **Ans.** $\frac{32}{105}$

25. Let A be the event that the family owns a colour television set and B be the event that the family owns a black and white television. Then, we have
$P(A) = 0.87$, $P(B) = 0.36$, $P(A \cap B) = 0.30$ **Ans.** 0.93

26. Solve as Example 12.
P (getting neither a doublet nor a total of 10)
$= 1 - P$ (getting a doublet or a total of 10) **Ans.** $\frac{7}{9}$

27. Solve as Example 14. **Ans.** $\frac{5}{9}$

28. Let $P(E)=\frac{5}{14}$. Then, $P(\bar{E})=1-P(E)=1-\frac{5}{14}=\frac{9}{14}$

Now,

(i) Odds in favour of occurrence of event $E=\frac{P(E)}{P(\bar{E})}$

Ans. $=5:9$

(ii) Odds against the occurrence of event $E=\frac{P(\bar{E})}{P(E)}$

Ans. $=\frac{9}{5}$

29. Let E_1, E_2, E_3 and E_4 be the events that the competitors A, B, C and D respectively win the competition.

Then, $P(E_1)=\frac{1}{3}$; $P(E_2)=\frac{1}{4}$; $P(E_3)=\frac{1}{5}$ and $P(E_4)=\frac{1}{6}$

Now, required probability

$=P(E_1 \cup E_2 \cup E_3 \cup E_4)$

$=P(E_1)+P(E_2)+P(E_3)+P(E_4)$ **Ans.** $=\frac{114}{120}$

30. Solve as Example 16. **Ans.** $\frac{4}{13}$

31. Solve as Example 20. **Ans.** $\frac{67}{200}$

32. Solve as Example 11. **Ans.** $\frac{3}{4}$

33. Solve as Example 13. **Ans.** $\frac{1}{4}$

34. Let A and B be the events that sum of numbers of two faces is divisible by 3 and 5 respective.

$\therefore A=\{(1, 2), (2, 1), (2, 4), (4, 2), (3, 3), (1, 5), (5, 1), (4, 5), (5, 4), (6, 3), (3, 6), (6, 6)\}$

and $B=\{(1, 4), (4, 1), (2, 3), (3, 2), (4, 6), (6, 4), (5, 5)\}$

and $A \cap B=\{\}$. Now, solve as Example 15. **Ans.** $\frac{17}{36}$

35. Let S be the sample space associated with the given random experiment. Then, $n(S)=36$.

(i) Let E be the event of getting the sum 5. Then,

$E=\{(1,4),(4,1),(2,3),(3,2)\}$

$\Rightarrow \quad n(E)=4$

$\therefore \quad P(E)=\frac{n(E)}{n(S)}=\frac{4}{36}=\frac{1}{9} \Rightarrow P(\bar{E})=1-\frac{1}{9}=\frac{8}{9}$

Hence, odds in favour of getting the sum $5=\frac{P(E)}{P(\bar{E})}$

Ans. 1:8

(ii) Let F be the event of getting the sum 6.

Then,

$F=\{(1,5),(5,1),(2,4),(4,2),(3,3)\} \Rightarrow n(F)=5$

$\therefore \; P(F)=\frac{n(F)}{n(S)}=\frac{5}{36} \Rightarrow P(\bar{F})=1-\frac{5}{36}=\frac{31}{36}$

Hence, odds against getting the sum $6=\frac{P(\bar{F})}{P(F)}$

Ans. 31:5

36. Let C be the event that the person will have his teeth cleaned. F be the event that the person will have a cavity filled. and E be the event that the person will have a tooth extracted.

Then, $P(C)=0.48$, $P(F)=0.25$, $P(E)=0.20$,

$P(C \cap F)=0.09$, $P(C \cap E)=0.12$, $P(E \cap F)=0.07$

and $P(C \cap F \cap E)=0.03$

Now, P (that a person will have at least one of these things)

$$=P(C \cup F \cup E)=P(C)+P(F)+P(E) -P(C \cap F)-P(F \cap E) -P(C \cap E)+P(C \cap F \cap E)$$

$$=0.48+0.25+0.20-0.09 -0.12-0.07+0.03=0.68$$

Ans. 0.68

37. Solve as Example 18.

Ans. (i) $\frac{1081}{812175}$ (ii) $\frac{47}{10829}$ (iii) $\frac{4606}{812175}$

38. Solve as Example 21. **Ans.** $\frac{3}{5}$

39. We have,

$P(A)+P(B)-2P(A \cap B)=1-x$ ...(i)

$P(B)+P(C)-2P(B \cap C)=1-2x$ ...(ii)

$P(C)+P(A)-2P(C \cap A)=1-x$ ...(iii)

and $P(A \cap B \cap C)=x^2$ ...(iv)

On adding Eqs. (i), (ii) and (iii), we get

$$P(A)+P(B)+P(C)-P(A \cap B)-P(B \cap C) -P(C \cap A)=\frac{3-4x}{2}$$

Now, $P(A \cup B \cup C)=\frac{3-4x}{2}+x^2=x^2-2x+\frac{3}{2}$

$$=(x-1)^2+\frac{1}{2}>\frac{1}{2}$$

40. Solve as Example 23. **Ans.** 0.556

41. Let A, B, C and D be the events that the horses P, Q, R and S respectively win the race.

Then, $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{4}$, $P(C)=\frac{1}{5}$ and $P(D)=\frac{1}{6}$.

Since, there can be only one winner.

$\therefore A$, B, C and D are mutually exclusive event.

Required probability $=P(A \cup B \cup C \cup D)$ **Ans.** $\frac{19}{20}$

42. Solve as Q. 41. **Ans.** $\frac{37}{60}$

43. Solve as Example 24. **Ans.** 0.27

44. Solve as Example 19. **Ans.** $\frac{4}{9}$

45. Solve as Example 27. **Ans.** 0.5

46. Solve as Example 31. **Ans.** (i) 0.60 (ii) 0.35

47. Let A = event that 1 red, 1 white and 2 black balls are drawn,

B = event that 1 red, 2 white and 1 black balls are drawn and

C = event that 2 red, 1 white and 1 black balls are drawn.

Here A, B and C are mutually exclusive events.

Hence, required probability

$$= P(A \cup B \cup C) = P(A) + P(B) + P(C) \quad \textbf{Ans. } \frac{48}{91}$$

48. (i) Number of faces with number '2' = 3

$$\therefore \quad P(2) = \frac{3}{6} = \frac{1}{2}$$

(ii) $P(1 \text{ or } 3) = (P \text{ not } 2) = 1 - P(2) = 1 - \frac{1}{2} = \frac{1}{2}$

(iii) Number of faces with number '3' = 1

$$\therefore \quad P(3) = \frac{1}{6}$$

$$\Rightarrow P(\text{not } 3) = 1 - P(3) = 1 - \frac{1}{6} = \frac{5}{6}$$

49. Given, total number of white balls = 5

and total number of black balls = 7

$\therefore$ Total number of balls = 12

Let S be the sample space associated with the given experiment. Then, $n(S) = {}^{12}C_4$.

Now, let E be the event of getting black balls only.

Then, $\quad n(E) = {}^7C_4$

$$\Rightarrow \quad P(E) = \frac{{}^7C_4}{{}^{12}C_4} = \frac{7 \times 6 \times 5 \times 4}{12 \times 11 \times 10 \times 9} = \frac{7}{99}$$

$$\Rightarrow \quad P(\bar{E}) = 1 - P(E) = 1 - \frac{7}{99} = \frac{92}{99}$$

Hence, odds against getting black balls

$$= \frac{P(\bar{E})}{P(E)} = \frac{\frac{92}{99}}{\frac{7}{99}} = \frac{92}{7} = 92:7$$

50. $A = \{21, 24, 27, 30, 33, 36, 39, 42, 45, 48\}$

$B = \{20, 25, 30, 35, 40, 45, 50\}$, $C = \{7, 14, 21, 28, 35, 42, 49\}$

Solve as Example 23. **Ans.** $\frac{19}{30}$

51. Let S be the sample space associated with the given random experiment. Then, $n(S) = 52$.

(i) Let E be the event of getting of face card.

Then, $\quad n(E) = 12$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

$$\Rightarrow \quad P(\bar{E}) = 1 - P(E) = 1 - \frac{3}{13} = \frac{10}{13}$$

Hence, odds in favour of getting a face card

$$= \frac{P(E)}{P(\bar{E})} = \frac{\frac{3}{13}}{\frac{10}{13}} = \frac{3}{10} = 3:10$$

(ii) Let F be the event of getting a spade. Then $n(F) = 13$.

$$\therefore \quad P(F) = \frac{n(F)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

$$\Rightarrow \quad P(\bar{F}) = 1 - P(F) = 1 - \frac{1}{4} = \frac{3}{4}$$

Hence, odds against of getting a spade

$$= \frac{P(\bar{F})}{P(F)} = \frac{\frac{3}{4}}{\frac{1}{4}} = \frac{3}{1} = 3:1$$

52. Solve as Example 33. **Ans.** $\frac{28363}{36315}$

53. We have, A and B are mutually exclusive events.

Also, it is given that $P(A) = 0.35$ and $P(B) = 0.45$

(i) $P(A') = 1 - P(A) = 1 - 0.35 = 0.65$

(ii) $P(B') = 1 - P(B) = 1 - 0.45 = 0.55$

(iii) $P(A \cup B) = P(A) + P(B) = 0.35 + 0.45 = 0.8$

[$\because$ A and B are mutually exclusive]

(iv) $P(A \cap B) = 0$

[$\because$ A and B are mutually exclusive, therefore $A \cap B = \phi$]

(v) $P(A \cap B') = P(A) - P(A \cap B)$

$= P(A) - 0 = P(A) = 0.35$

(vi) $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

54. Let A and B denote the events that, the selected students opted NCC and NSS, respectively.

Given, $\quad n(A) = 30$, $n(B) = 32$

$n(A \cap B) = 24$, $n(S) = 60$

[$\because$ 24 students opted for both NCC and NSS i.e. they are common in both]

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{30}{60}, \; P(B) = \frac{n(B)}{n(S)} = \frac{32}{60}$$

$$\text{and} \quad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{24}{60}$$

(i) P(student opted for NCC or NSS)

$$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{30}{60} + \frac{32}{60} - \frac{24}{60}$$

$$= \frac{30 + 32 - 24}{60} = \frac{62 - 24}{60}$$

$$= \frac{38}{60} = \frac{19}{30}$$

(ii) P(student opted neither NCC nor NSS)

$$= P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$$

$$= 1 - P(A \cup B) = 1 - \frac{19}{30} = \frac{30 - 19}{30} = \frac{11}{30}$$

(iii) P(student has opted NSS but not NCC)

$$= P(B \cap \bar{A}) = P(B) - P(A \cap B) = \frac{32}{60} - \frac{24}{60} = \frac{8}{60} = \frac{2}{15}$$

SUMMARY

- Every subset of a sample space, is an **event**.
- **Types of Event**
  (i) Whole space $S \subseteq S$ and it represents an event, which is called **sure event** or certain event.
  (ii) Empty set $\phi \subseteq S$ and it represents an **impossible event**.
  (iii) An event having only one sample point of S, is called **simple** or **elementary event**.
  (iv) An event having more than one sample point, is called a **compound event**.
- Events are said to be **equally likely**, if none of them is expected to occur in preference to the other.
- A set of events $(E_1, E_2, \ldots, E_n)$ is said to be **exhaustive**, if one of them is necessarily occurs whenever the experiment is performed. i.e. $E_1 \cup E_2 \cup \ldots \cup E_n = S$
- Two or more events are said to be **mutually exclusive**, if no two of them can occur together. i.e. for events E_1 and E_2, $E_1 \cap E_2 = \phi$.
- The chances that a particular event will occur when we perform an experiment, is called the **probability of occurrence** of that event.
- **Algebra of Events**
  (i) E' or not E, is called the **complementary event** to E. i.e. Not $E = E' = S - E$.
  (ii) Let E_1 and E_2 be two events associated with S, then
    (a) the event either E_1 or E_2 or both $= E_1 \cup E_2$.
    (b) the event E_1 and $E_2 = E_1 \cap E_2$.
    (c) the event E_1 but not $E_2 = E_1 - E_2$ or $E_1 \cap E_2'$.
- **Axiomatic Approach to Probability**
  Let S be a sample space containing outcomes $E_1, E_2, \ldots, E_n$. Then, from the axiomatic definition of probability, we have
  (i) $0 \le P(E_i) \le 1, \forall E_i \in S$
  (ii) $P(E_1) + P(E_2) + \ldots + P(E_n) = 1$
  (iii) For any event A, $P(A) = \sum_{i=1}^{n} P(E_i), E_i \in A$
- **Probability of an Event**
  Let S be the sample space and E be an event, then $P(E) = \dfrac{\text{Number of elements in set } E}{\text{Number of elements in set } S} = \dfrac{n(E)}{n(S)}$
- Probability of Event A or B is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Let E be any event and $\overline{E}$ be its complementary event. Then, $P(\overline{E}) = 1 - P(E)$.

CHAPTER PRACTICE

OBJECTIVE TYPE QUESTIONS

1. Which of the following is true?

I. The empty set ϕ is called an impossible event.

II. The whole sample space S is called the sure event.

(a) Only I is true
(b) Only II is true
(c) Both I and II are true
(d) Both I and II are false

2. Consider the experiment of rolling a die. Let A be the event 'getting a prime number' and B be the event 'getting an odd number'.

Then, which of the following is true?

I. A or $B = A \cup B = \{1, 2, 3\}$

II. A and $B = A \cap B = \{3, 5\}$

III. A but not $B = A - B = \{2\}$

IV. Not $A = A' = \{1, 5, 6\}$

(a) Only I is true (b) Only II is true
(c) II and III are true (d) Only IV is true

3. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Then, E and F are

(a) mutually exclusive
(b) exhaustive
(c) mutually exclusive and exhaustive
(d) None of the above

4. Seven persons are to be seated in a row. The probability that two particular persons sit next to each other, is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{2}{7}$ (d) $\frac{1}{2}$

5. A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, then the probability that it is rusted or is a nail, is ...K... . Here, K refers to

(a) $\frac{3}{16}$ (b) $\frac{5}{16}$ (c) $\frac{11}{16}$ (d) $\frac{14}{16}$

6. 4 cards are drawn from a well-shuffled deck of 52 cards. Thus, the probability of obtaining 3 diamonds and one spade is

(a) $\frac{{}^{13}C_1 \times {}^{12}C_1}{{}^{52}C_1}$ (b) $\frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4}$

(c) $\frac{{}^{13}C_1 \times {}^{13}C_2}{{}^{52}C_2}$ (d) $\frac{{}^{13}C_3 \times {}^{13}C_2}{{}^{51}C_4}$

7. P and Q are two condidates seeking admission in NEET. The probability that P is selected is 0.5 and the probability that both P and Q are selected is at most 0.3. Then, the probability of Q being selected is

(a) ≤ 0.8 (b) ≥ 0.8
(c) $= 0.8$ (d) None of these

8. If A and B are any two events such that $P(A \cup B) = \frac{1}{2}$ and $P(\overline{A}) = \frac{2}{3}$, then $P(\overline{A} \cap B) =$

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$

VERY SHORT ANSWER Type Questions

9. A pair of dice is rolled. If the outcome is a doublet, a coin is tossed. Determine the total number of elementary events associated to this experiment.

10. An experiment has four possible outcomes A, B, C and D, that are mutually exclusive. Explain why the following assignments of probabilities are not permissible?

(i) $P(A) = 0.12, P(B) = 0.63, P(C) = 0.45, P(D) = -0.20$

(ii) $P(A) = \frac{9}{120}, P(B) = \frac{45}{120}, P(C) = \frac{27}{120}, P(D) = \frac{46}{120}$

11. A bag contains 9 red and 12 white balls. One ball is drawn at random. Find the probability that the ball drawn is red.

12. In a lottery, there are 10 prizes and 25 blanks. Find the probability of getting a prize.

SHORT ANSWER Type I Questions

13. (i) How many two-digit positive integers are multiple of 3?

(ii) What is the probability that a randomly chosen two-digit positive integer is a multiple of 3?

14. What is the probability that a leap year selected at random will contain 53 Sunday?

15. Five marbles are drawn from a bag which contains 7 blue marbles and 4 black marbles. What is the probability that

(i) all will be blue?

(ii) 3 will be blue and 2 black?

16. If A and B are two events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$, then find the value of $P(A \cap \overline{B})$.

SHORT ANSWER Type II Questions

17. A bag contains 6 discs of which 4 are red, 3 are blue and 2 are yellow. The discs are similar in shape and size. A disc is drawn at random from the bag. Calculate the probability that it will be

(i) red (ii) yellow (iii) blue (iv) not blue

(v) either red or blue. [NCERT]

18. One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will be

(i) a diamond (ii) an ace

(iii) a black card (iv) not a diamond

(v) not an ace (vi) not a black card

(vii) a red card. [NCERT Exemplar]

19. Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that

(i) both Anil and Ashima will not qualify the examination.

(ii) atleast one of them will not qualify the examination.

(iii) only one of them will qualify the examination. [NCERT]

20. In a single throw of three dice, find the probability of getting a total of 17 or 18.

LONG ANSWER Type Questions

21. What is the probability that a non-leap year selected at random will contain 53 Tuesday or 53 Wednesday?

22. A team of medical students doing their internship have to assist during surgeries at a city hospital. The probabilities of surgeries rated as very complex, routine, simple rated as very complex, complex, complex, routine, simple or very simple respectively 0.15, 0.20, 0.31, 0.26 and 0.08. Find the probabilities that a particular surgery will be rated

(i) complex or very complex.

(ii) neither very complex nor very simple.

(iii) routine or complex.

(iv) routine or simple. [NCERT Exemplar]

23. From the employees of a company, 5 persons are elected to represent them in the managing committee of the company. Particulars of the five persons are as follows: [NCERT]

S.No.	Person	Age (in yr)
1	Male	30
2	Male	33
3	Female	46
4	Female	28
5	Male	41

A person is selected at random from this group to act as a spokespersons. What is the probability that a spokesperson will be either male or over 35 yr?

24. One of the four persons John, Rita, Aslam or Gurpreet will be promoted next month. Consequently, the sample space consists of four elementary outcomes S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted} You are told that the chances of John's promotion is same as that of Gurpreet, Rita's chances of promotion are twice as likely as Johns. Aslam's chances are four times that of John.

(i) Determine P (John promoted)

P (Rita promoted);

P (Aslam promoted)

P (Gurpreet promoted)

(ii) If A = {John promoted or Gurpreet promoted}, then find $P(A)$. [NCERT Exemplar]

CASE BASED Questions

25. There are 4 red, 5 blue and 3 green marbles in a basket.

Based on the above information answer the following.

(i) If two marbles are picked at randomly, then the probability that both red marbles is

(a) $\frac{3}{7}$ (b) $\frac{1}{2}$ (c) $\frac{1}{11}$ (d) $\frac{1}{6}$

(ii) If three marbles are picked at randomly, then the probability that all green marbles is

(a) $\frac{1}{220}$ (b) $\frac{1}{55}$ (c) $\frac{1}{75}$ (d) $\frac{1}{80}$

(iii) If two marbles are picked at randomly then find the probability that both are not blue marbles.

(a) $\frac{6}{11}$ (b) $\frac{5}{12}$ (c) $\frac{7}{22}$ (d) $\frac{9}{11}$

(iv) If three marbles are picked at randomly, then the probability that atleast one of them is blue, is

(a) $\frac{7}{12}$ (b) $\frac{37}{44}$ (c) $\frac{5}{12}$ (d) $\frac{1}{44}$

(v) If three marbles are picked randomly, then the probability that either all are red or all are green

(a) $\frac{7}{44}$ (b) $\frac{7}{12}$ (c) $\frac{5}{12}$ (d) $\frac{1}{44}$

22. Two students Anil and Vijay appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Vijay will qualify is 0.10. The probability that both will qualify is 0.02.

On the basis of above information, answer the following questions.

(i) Find the probability that Vijay will not qualify the exam.

(a) 0.9 (b) 0.5 (c) 0.8 (d) 0.2

(ii) Find the probability that atleast one of them will qualify the exam.

(a) 0.09 (b) 0.13 (c) 0.25 (d) 0.19

(iii) Find the probability that atleast one of them will not qualify the exam.

(a) 0.82 (b) 0.74 (c) 0.56 (d) 0.98

(iv) Find the probability that both Anil and Vijay will not qualify the exam.

(a) 0.43 (b) 0.67 (c) 0.87 (d) 0.91

(v) Find the probability that only one of them will qualify the exam.

(a) 0.21 (b) 0.11 (c) 0.31 (d) 0.25

27. On her vacation, Veena visits four cities. Delhi, Lucknow, Agra, Meerut in a random order.

On the basis of above information, answer the following questions.

(i) What is the probability that she visits Delhi before Lucknow?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

(ii) What is the probability she visit Delhi before Lucknow and Lucknow before Agra?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

(iii) What is the probability she visits Delhi first and Lucknow last?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{12}$

(iv) What is the probability she visits Delhi either first or second?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$

(v) What is the probability she visits Delhi just before Lucknow?

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{1}{12}$

28. Four friends Daksh, Yash, Sourabh, and Raju are playing cards. Daksh, shuffling a cards and told to Raju choose any four cards.

On the basis of above information, answer the following questions.

(i) What is the probability that Raju getting all face card.

(a) $\frac{^{12}C_4}{^{52}C_4}$ (b) $\frac{^{16}C_4}{^{52}C_4}$

(c) $\frac{(^{12}C_2)^2}{^{52}C_4}$ (d) None of these

(ii) What is the probability that Raju getting two red cards and two black card.

(a) $\frac{(^{13}C_2)^2}{^{52}C_4}$ (b) $\frac{^{26}C_4}{^{52}C_4}$

(c) $\frac{(^{26}C_2)^2}{^{52}C_4}$ (d) $\frac{(^{26}C_4)^2}{^{52}C_4}$

(iii) What is the probability that Raju getting one card from each suit.

(a) $\frac{^{13}C_4}{^{52}C_4}$ (b) $\frac{(13)^4}{^{52}C_4}$

(c) $\frac{(^{13}C_4)^2}{^{52}C_4}$ (d) None of these

(iv) What is the probability that Raju getting all king cards.

(a) $\frac{1}{^{52}C_4}$ (b) $\frac{2}{^{52}C_4}$

(c) $\frac{4}{^{52}C_4}$ (d) $\frac{6}{^{52}C_4}$

(v) What is the probability that Raju getting two king and two Jack cards.

(a) $\frac{^{4}C_2}{^{52}C_4}$ (b) $\frac{36}{^{52}C_4}$

(c) $\frac{6}{^{52}C_4}$ (d) None of these

HINTS & ANSWERS

1. (c) The empty set ϕ is called an impossible event and S, i.e. the whole sample space is called the sure event.

2. (c) Here, $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3, 5\}$ and $B = \{1, 3, 5\}$

Obviously,

I. 'A or B' = $A \cup B = \{1, 2, 3, 5\}$

II. 'A and B' = '$A \cap B = \{3, 5\}$

III. 'A but not B' = $A - B = \{2\}$

IV. 'not A' = $A' = \{1, 4, 6\}$

3. (d) Let E = The die shows 4 = $\{4\}$

Let F = The die shows even number = $\{2, 4, 6\}$

$\therefore\ E \cap F = \{4\} \neq \phi$

Also, $E \cup F \neq \{1, 2, 3, 4, 5, 6\} = 5$

Hence, E and F are neither mutually exclusive nor exhaustive.

4. (c) Given, number of persons = 7

Total number of sitting arrangements = 7!

Favourable number of arrangements = 6!

2 persons can be arranged in two ways.

Total number of favourable arrangments = 2(6!)

$\therefore$ Required probability $= \frac{2(6!)}{7!} = \frac{2}{7}$

5. (c) Given that, number of nails = 6

Number of nuts = 10

Total number of items = 6 + 10 = 16

and half of them are rusted

$\therefore$ Total rusted items = 3 + 5 = 8

$P(\text{the items is rusted or nail}) = \frac{8}{16} + \frac{6}{16} - \frac{3}{16} = \frac{11}{16}$

6. (b) Total number of ways selecting 4 cards out of 52 cards

$$= {}^{52}C_4$$

If E be the event obtaining 3 diamonds and 1 spade, then

$$n(E) = {}^{13}C_3 \times {}^{13}C_1$$

$\therefore$ Required probability $= \frac{^{13}C_3 \times {}^{13}C_1}{^{52}C_4}$

7. (a) Let A_1 and A_2 be two events defined as follows

$A_1 = P$ is selected, $A_2 = Q$ is selected

We have, $P(A_1) = 0.5$ and $P(A_1 \cap A_2) \leq 0.3$

Now, $P(A_1 \cup A_2) \leq 1$

$\Rightarrow\ P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq 1$

$\Rightarrow\ P(A_2) \leq 0.5 + P(A_1 \cap A_2)$

$\Rightarrow\ P(A_2) \leq 0.8$

8. (c) Clearly, $\overline{A} \cap B$ and A are mutually exclusive events such that

$$A \cup B = A \cup (\overline{A} \cap B)$$

$\therefore\ P(A \cup B) = P(A) + P(\overline{A} \cap B)$

$$\left(\because P(A \cup B) = \frac{1}{2}, P(\overline{A}) = \frac{2}{3}\right)$$

$\Rightarrow\ P(\overline{A} \cap B) = \frac{1}{6}$

9. 42

10. (i) $P(D)$ is not possible (ii) $P(S) \neq 1$

11. $\frac{3}{7}$

12. $\frac{2}{7}$

13. (i) 30 (ii) $\frac{1}{3}$

14. $\frac{2}{7}$

15. (i) $\frac{1}{22}$ (ii) $\frac{5}{11}$

16. 0.2

17. (i) $\frac{4}{9}$ (ii) $\frac{2}{9}$ (iii) $\frac{1}{3}$ (iv) $\frac{2}{3}$ (v) $\frac{7}{9}$

18. (i) $\frac{1}{4}$ (ii) $\frac{1}{13}$ (iii) $\frac{1}{2}$ (iv) $\frac{3}{4}$ (v) $\frac{12}{13}$ (vi) $\frac{1}{2}$ (vii) $\frac{1}{2}$

19. (i) 0.87 (ii) 0.98 (iii) 0.11

20. $\frac{1}{54}$ **21.** $\frac{2}{7}$

22. (i) 0.35 (ii) 0.77 (iii) 0.51 (iv) 0.57

23. $\frac{4}{5}$

24. (i) P (John promoted) $=\frac{1}{8}$; P (Rita promoted) $=\frac{1}{4}$;

P (Aslam promoted) $=\frac{1}{2}$; P (Gurpreet promoted) $=\frac{1}{8}$

(ii) $P(A)=\frac{1}{4}$

25. Total marbles $= 4 + 5 + 3 = 12$

(i) (c) Required probability $=\frac{{}^4C_2}{{}^{12}C_2}=\frac{\frac{4\times3}{2\times1}}{\frac{12\times11}{2\times1}}=\frac{1}{11}$

(ii) (a) Required probability $=\frac{{}^3C_3}{{}^{12}C_3}=\frac{1}{\frac{12\times11\times10}{3\times2}}=\frac{1}{220}$

(iii) (c) Required probability $=\frac{{}^7C_2}{{}^{12}C_2}=\frac{\frac{7\times6}{2\times1}}{\frac{12\times11}{2\times1}}=\frac{21}{66}=\frac{7}{22}$

(iv) (b) Required probability $=1-P$ (None is blue)

$$=1-\frac{{}^7C_3}{{}^{12}C_3}$$

$$=1-\frac{\frac{7\times6\times5}{3\times2}}{\frac{12\times11\times10}{3\times2}}$$

$$=1-\frac{7}{44}=\frac{37}{44}$$

(v) (d) Required probability $=\frac{{}^3C_3+{}^4C_3}{{}^{12}C_3}$

$$=\frac{1+4}{220}=\frac{5}{220}=\frac{1}{44}$$

26. Let E_1 and E_2 denotes the events that Anil and Vijay will respectively qualify the exam.

(i) (a) The probability that Vijay will not qualify the exam

$$=1-P(E_2)$$
$$=1-0.\ 10=0.9$$

(ii) (b) $P(E_1\cup E_2)=P(E_1)+P(E_2)-P(E_1\cap E_2)$

$$=0.05+0.10-0.02=0.13$$

(iii) (d) Probability of atleast one of them does not qualify

$$=P(E'_1\cup E'_2)=P((E_1\cap E_2)')$$
$$=1-P(E_1\cap E_2)=1-0.02=0.98$$

(iv) (c) Probability that both Anil and Vijay will not qualify the exam

$$=P(E'_1\cap E'_2)=P((E_1\cup E_2)')$$
$$=1-P(E_1\cup E_2)=1-0.13=0.87$$

(v) (b) Probability that only one of them will qualify the exam

$$=P((E_1-E_2)\cup(E_2-E_1))$$
$$=P(E_1-E_2)+P(E_2-E_1)$$
$$=P(E_1\cup E_2)-P(E_1\cap E_2)$$
$$=0.13-0.02=0.11$$

27. Let the Veena visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D. Number of way's in which Veena can visit four cities A, B, C and D is $4!$ i.e. 24.

$\therefore\ n(S)=24$

Clearly, sample space for this experiment is

$$S=\begin{Bmatrix} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB,\\ BACD, BADC, BCAD, BCDA, BDAC, BDCA,\\ CABD, CADB, CBAD, CBDA, CDAB, CDBA,\\ DABC, DACB, DCAB, DCBA, DBAC, DBCA \end{Bmatrix}$$

(i) (a) Let E_1 be the event that Veena visits A before B. Then,

$$E_1=\begin{Bmatrix} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB,\\ CABD, CADB, CDAB, DABC, DACB, DCAB \end{Bmatrix}$$

$\Rightarrow n(E_1)=12$

$\therefore$ P(she visits A before B) $=P(E_1)=\frac{n(E_1)}{n(S)}=\frac{12}{24}=\frac{1}{2}$

(ii) (c) Let E_2 be the event that she visits A before B and B before C.

Then, $E_2=\{ABCD, ABDC, ADBC, DABC\}$

$\Rightarrow\ n(E_2)=4$

$\therefore\ P$ (she visits A before B and B before C) $=P(E_2)$

$$=\frac{n(E_2)}{n(S)}=\frac{4}{24}=\frac{1}{6}$$

(iii) (d) Let E_3 be the event that she visits A first and B last.

Then, $E_3=\{ACDB, ADCB\}$

$n(E_3)=2$

$\therefore\ P$ (she visits A first and B last) $=P(E_3)$

$$=\frac{n(E_3)}{n(S)}=\frac{2}{24}=\frac{1}{12}$$

(iv) (c) Let E_4 be the event that she visits A either first or second. Then,

$$E_4=\begin{Bmatrix} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB,\\ BACD, BADC, CABD, CADB, DABC, DACB \end{Bmatrix}$$

$\Rightarrow\ n(E_4)=12$

Hence, P (she visits A either first or second)

$$= P(E_4) = \frac{n(E_4)}{n(S)} = \frac{12}{24} = \frac{1}{2}$$

(v) (a) Let E_5 be the event that she visits A just before B. Then,

$$E_5 = \{ABCD, ABDC, CABD, CDAB, DABC, DCAB\}$$

$\Rightarrow n(E_5) = 6$

Hence, P (she visits A just before B)

$$= P(E_5) = \frac{n(E_5)}{n(S)} = \frac{6}{24} = \frac{1}{4}$$

28. Total number of possible outcomes $= {}^{52}C_4$

(i) (a) We know that there are 12 face cards

$\therefore$ Number of favourable outcomes $= {}^{12}C_4$

$\therefore$ Required probability $= \frac{{}^{12}C_4}{{}^{52}C_4}$

(ii) (c) We know that there are 26 red and 26 black cards

$\therefore$ Number of favourable outcomes $= {}^{26}C_2 \times {}^{26}C_2$

$\therefore$ Required probability $= \frac{({}^{26}C_2)^2}{{}^{52}C_4}$

(iii) (b) There are 4 suits each having 13 cards

$\therefore$ Number of favourable outcomes $= ({}^{13}C_1)^4$

$\therefore$ Required probability $= \frac{(13)^4}{{}^{52}C_4}$

(iv) (a) There are 4 king

$\because$ Number of favourable outcomes $= {}^4C_4 = 1$

$\therefore$ Required probability $= \frac{1}{{}^{52}C_4}$

(v) (b) In playing cards there are 4 king and 4 jack cards.

$\because$ Number of favourable outcomes $= ({}^4C_2 \times {}^4C_2)$

$= 6 \times 6 = 36$

$\therefore$ Required probability $= \frac{36}{{}^{52}C_4}$